# X-ray Clusters: Spatial-Spectral Modeling

Paul Nulsen

Harvard-Smithsonian Center for Astrophysics

# Clusters and Groups of Galaxies

Collections of up thousands of galaxies, bound by gravity and dynamically stable (collapsed & *virialized).*

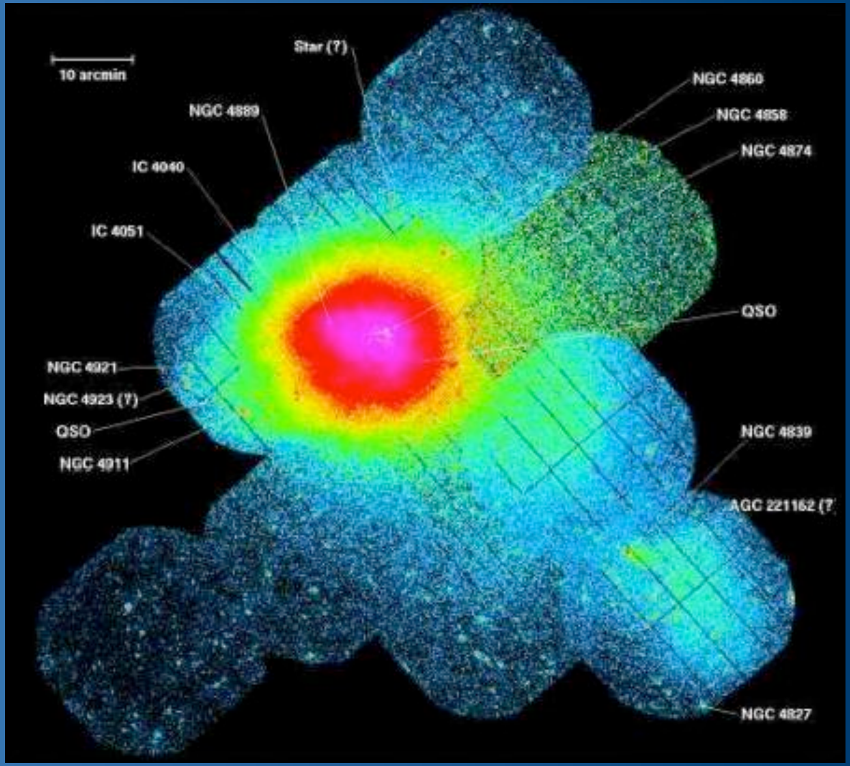
Roughly spherical.

Masses up to $\approx 10^{15}\ M_{\odot}$.

In rich (massive) clusters, most baryons form a diffuse hot atmosphere filling the cluster.

Gas densities: electron density $n_e \sim 10^{-4} - 0.1\ cm^{-3}$.

Note: mean baryon density within the virial radius $\approx 10^{-4}\ cm^{-3}$.

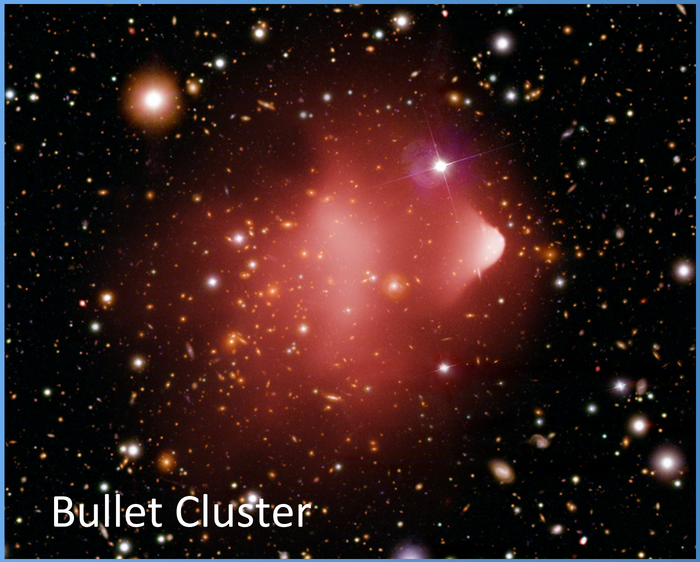



Coma Cluster (XMM-Newton)

Gas in hot atmospheres of isolated galaxies, groups and clusters is close the thermal equilibrium – emits optically thin thermal radiation (bremsstrahlung plus lines).

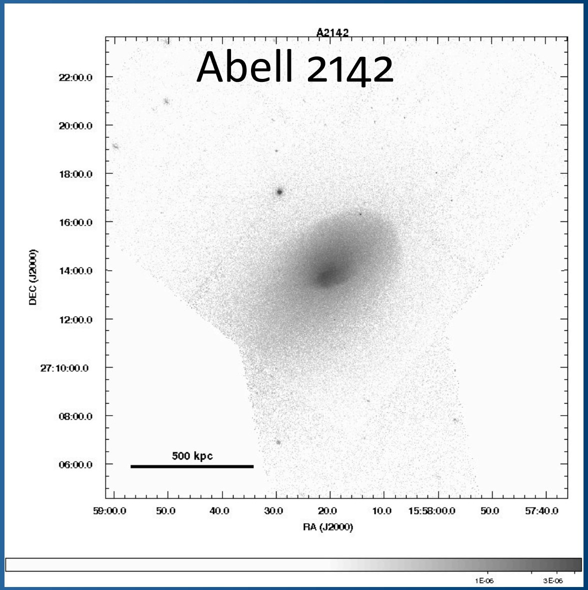
Temperatures: $T \sim 10^7 - 10^8$ K, or $kT \sim 1 - 10$ keV.

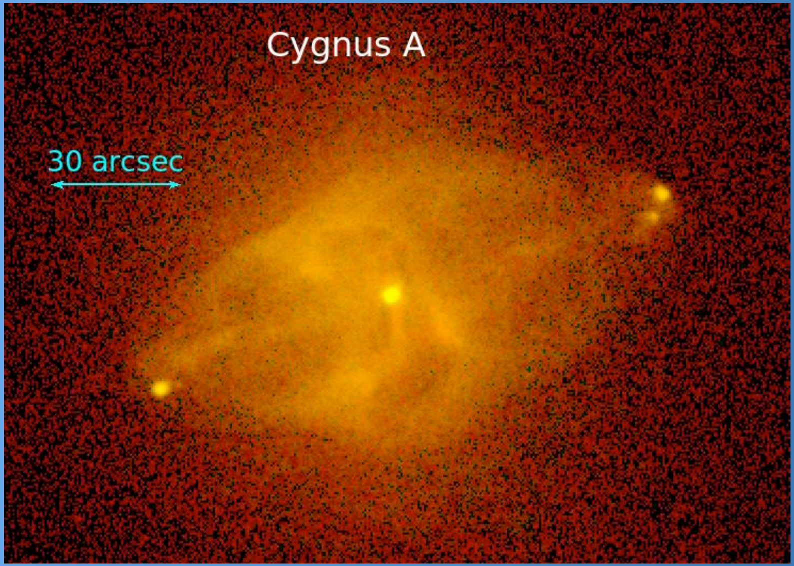
# Structure in Clusters and Groups

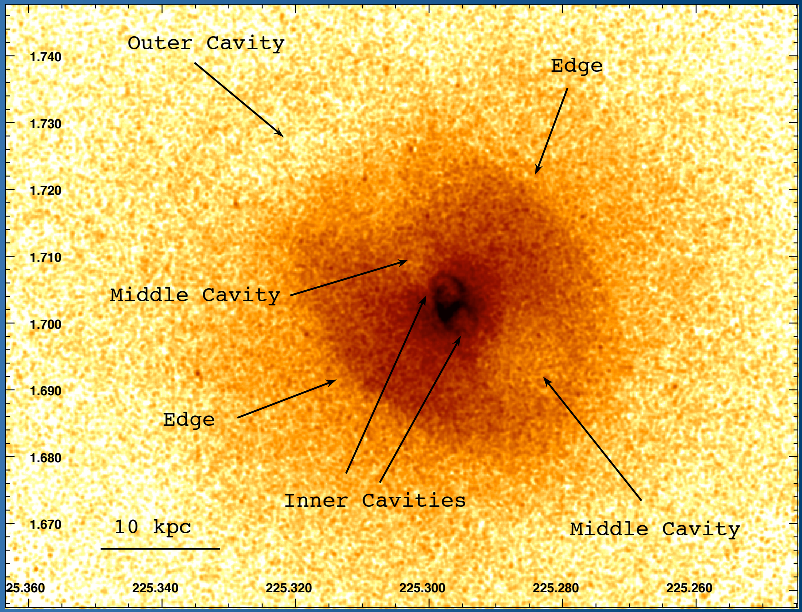



Merger shocks (rare), cold fronts and AGN outbursts add structure to clusters.

Large-scale symmetry is only broken by major mergers and binary clusters.




NGC 5813

# Measuring Gas Properties

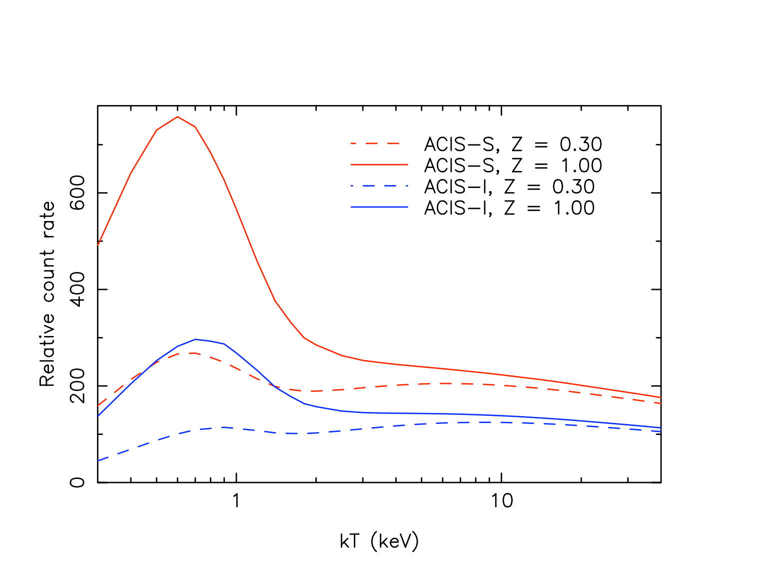
Depending on element abundances, for kT > 1 – 2 keV, broadband count rate is insensitive to temperature.

*Also* count rate is proportional to emission measure

$$\int_V n_e n_H dV$$

so that density estimates scale as the square root of count rate and they are insensitive to temperature.

*Chandra* 0.5 – 7 keV count rate vs temperature




Gas temperatures are determined by fitting spectra – requires many more photons.

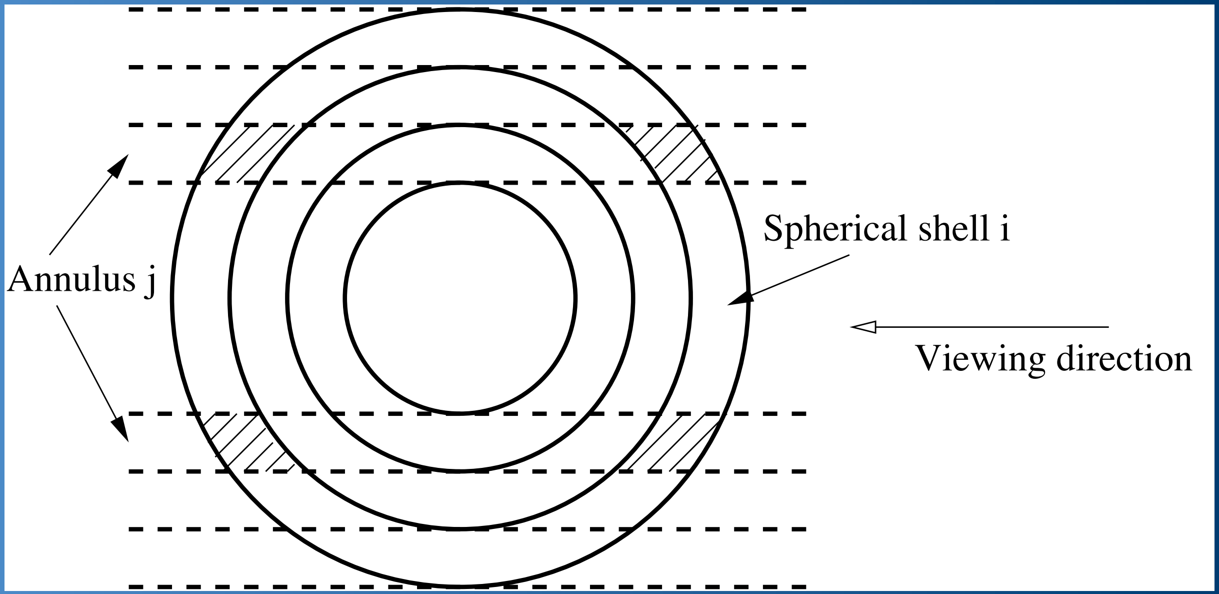
Abundances are also determined from spectra – more challenging still.

# The Deprojection Model

Spectra are extracted from concentric annuli.

Emission from gas with a range of densities, temperatures and abundances is projected into each annulus.

Divide the 3-d volume into spherical (ellipsoidal) shells, one corresponding to each annulus.




Gas in each spherical shell is assumed to be uniform (temperature, density & abundances).

Spectrum for annulus *j* is a sum of the models for shell *j* and the shells that surround it.

If the model norm for shell *i* is for its whole volume, $V_i$, the weight for shell *i* in annulus *j* would be $W_{i,j} = V_{i,j} / V_i$., where $V_{i,j}$ is the volume in the intersection of shell *i* and cylinder *j*.

This form is used in the XSPEC model “projct” and the Sherpa model “deproject.”

# X-ray Mass Determinations

Rely on the X-ray emitting gas being in hydrostatic equilibrium, in a spherically symmetric gravitational potential (and gas pressure is dominant).

Equation of hydrostatic equilibrium:

$$\frac{dp}{dr} = -\rho \frac{GM(r)}{r^2} \qquad \Rightarrow \qquad M(r) = -\frac{kTr}{\mu m_{\mathrm{H}} G}\left(\frac{d\ln T}{d\ln r} + \frac{d\ln n}{d\ln r}\right)$$

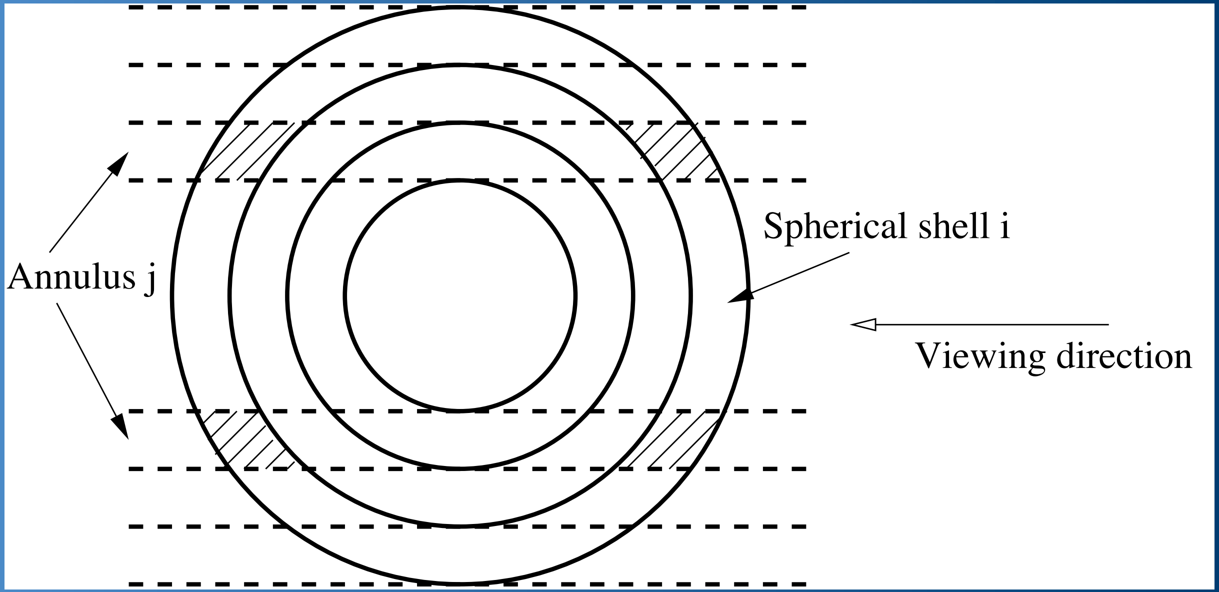
Direct application would require us to differentiate discrete data and the derivatives would be a major source of error.

In practice, additional, model-dependent assumptions are used, e.g., an analytic form is assumed for the gas density, n(r), and the temperature, T(r).

Model-dependent assumptions make masses appear more accurate than warranted.

They are not necessary with high quality data, e.g. Nulsen & Böhringer (1995), Arabadjis et al. (2004).

# A Physical Model for Mass determinations

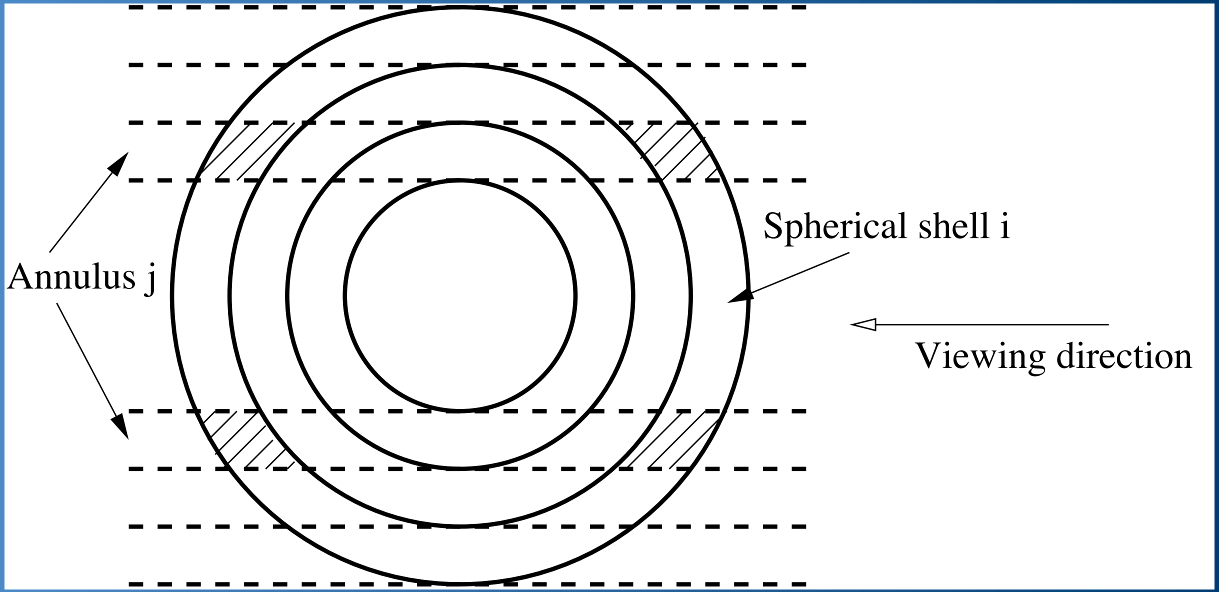



In addition to spherical symmetry and hydrostatic equilibrium, assume:

1. The gas in each spherical shell is isothermal
2. The gravitating matter density is uniform in each spherical shell

With many shells, this model can approximate any real distribution of gravitating mass and gas temperature – loosely "model-independent"

# A Physical Model for Mass determinations



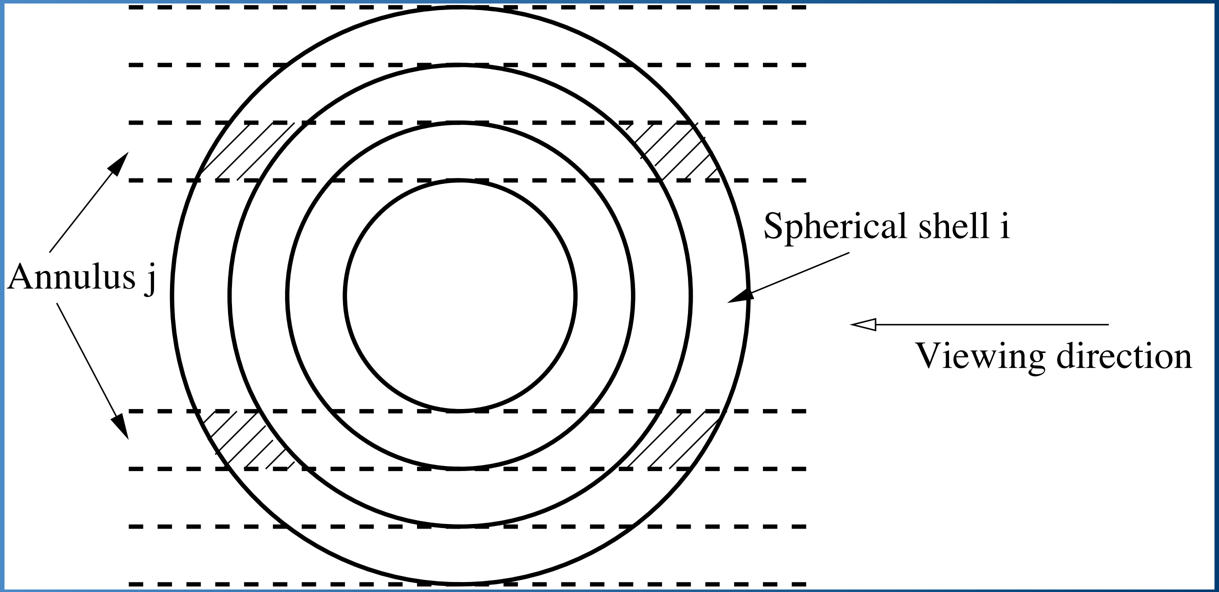

1. The gas in each spherical shell is isothermal

Electron density within shell *i* is then $$n_e(r) = n_{e,i} \exp\left[-\frac{\mu m_H\{\Phi(r) - \Phi(r_i)\}}{kT_i}\right], \quad r_i < r < r_{i+1}$$

Pressure continuity requires $$n_{e,i+1} T_{i+1} = n_{e,i} \exp\left[-\frac{\mu m_H\{\Phi(r_{i+1}) - \Phi(r_i)\}}{kT_i}\right] T_i$$

– once the potential, *Φ(r),* and temperatures are specified, the electron density is determined up to a single scale factor, $n_{e,1}$

# A Physical Model for Mass determinations

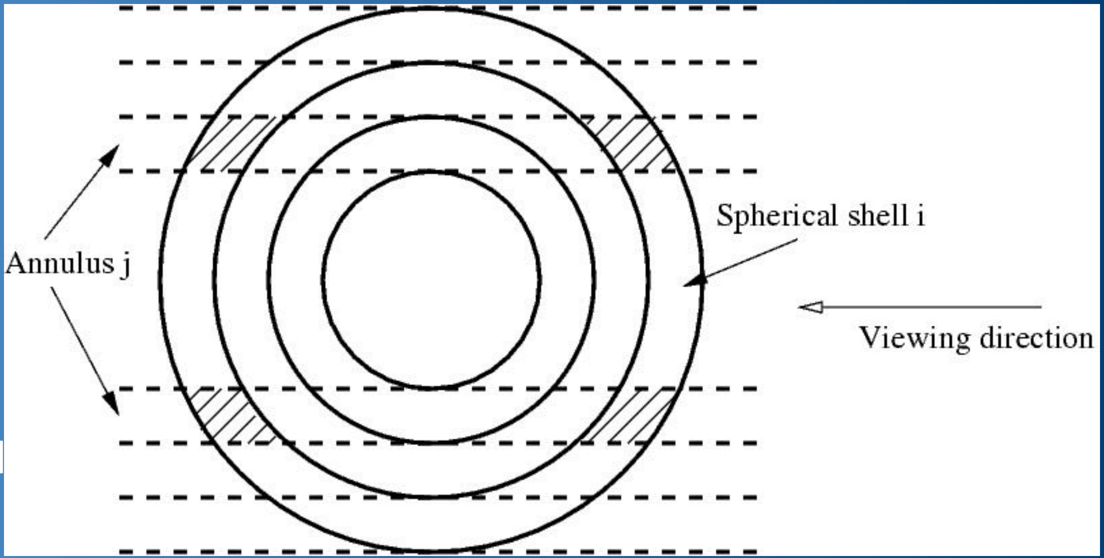



If the gravitating matter density in each spherical shell is constant

Gravitational potential $$\Phi(r) - \Phi(r_i) = \int_{r_i}^{r} \frac{GM(r)}{r^2} dr = \left[ \frac{GM_i}{r_i} + \frac{2\pi}{3} G\rho_i (r - r_i)(r + 2r_i) \right] \frac{r - r_i}{r}$$

Model parameters (temperatures, $T_i$, and gravitating matter densities, $\rho_i$) determine gas density distribution, up to a single scale factor.

# X-ray Spectrum




Emission measure of gas from shell *i* seen in annulus *j* (hatched region):

$$\mathrm{EM}_{i,j} = \int_{\mathrm{Intersection}} n_{\mathrm{e}} n_{\mathrm{H}} dV = 4\pi \int_{r_i}^{r_{i+1}} n_{\mathrm{e}}(r) n_{\mathrm{H}}(r) \left( \sqrt{r^2 - r_j^2} - \sqrt{r^2 - r_{j+1}^2} \right) r dr$$

Spectrum from annulus *j* has the form $F_j(E) = \sum_{i=j}^{n} \mathrm{EM}_{i,j} f(E, T_i, A_i)$

where $f(E,T_i,A_i)$ is the spectrum for unit emission measure.

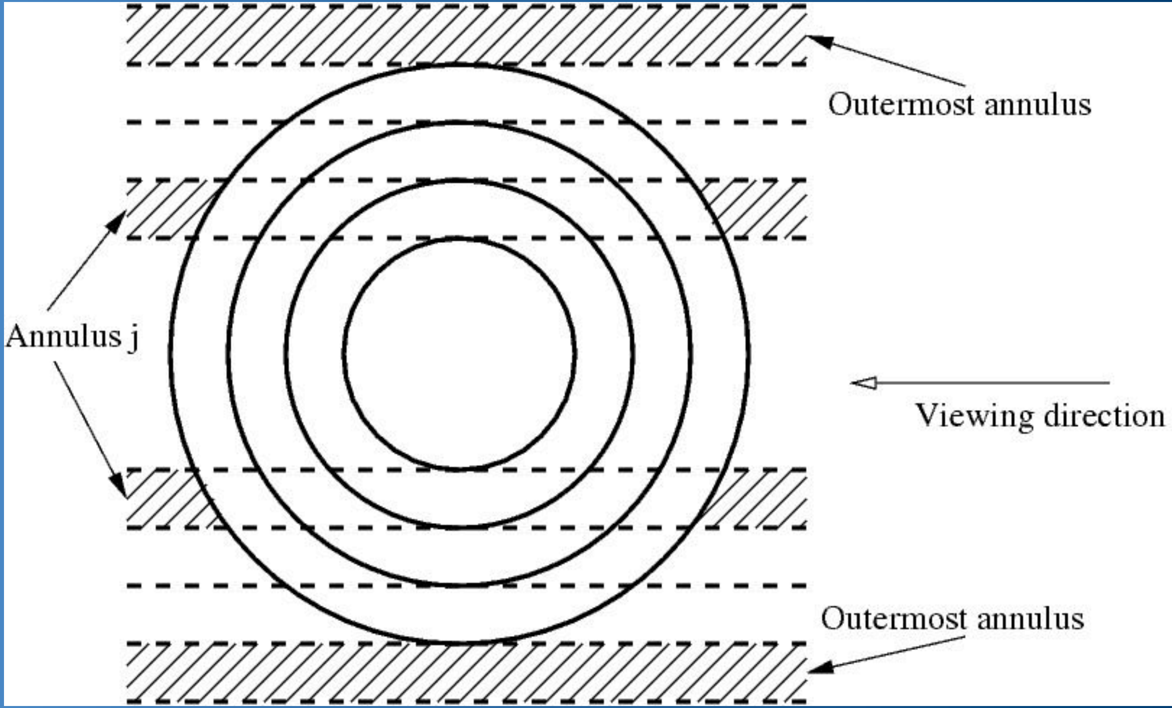
Implemented as an XSPEC mixing model, CLMASS $F_j(E) = K \sum_{i=j}^{n} \mathrm{W}_{i,j} f(E, T_i, A_i)$

with weights $W_{i,j} = \mathrm{EM}_{\mathrm{i,j}} / \mathrm{EM}_{1,1}$

=> only the norm for the innermost shell is free.

# Cluster Background

If cluster emission extends beyond outermost annulus, it contributes a dfferent amount to each inner annulus.




One approach:

Assume gas outside shells follows beta model, $n(r) = n_o(1 + r^2/a^2)^{-3\beta/2}$, and use spectrum of the outermost annulus to determine its temperature, normalization, etc.

Included as an option for the XSPEC mixing model

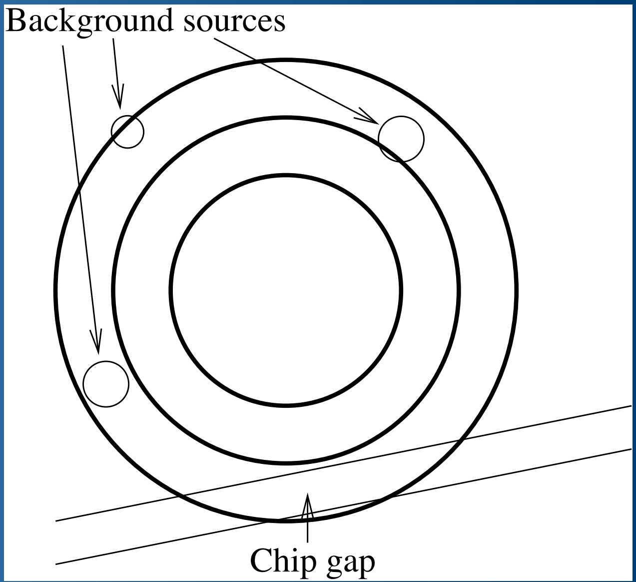
– not important if the data cover a cluster completely.

# Gory, but Essential Detail

Deprojection and mass models depend on relative normalization of the annular spectra.

Some area is always lost: e.g., bad pixels, chip edges, chip gaps and background sources.

In Sherpa, this can be correction by multiplying the model for each annulus by a factor
f = (exposed area) / (geometric area of annulus).




For XSPEC mixing models, the correction must be applied to AREASCAL, or to the angular ranges used by projct and the mass models (XFLT keywords).

To Compute the correction, make an exposure map, omitting effective area and QE (see mkinstmap and mkexpmap)
– summing the exposure map for an annular region and dividing by the exposure time and geometric area of the annulus (in pixels) gives the appropriate correction (approximately).

# A Cluster Sample

CLMASS model applied to *CHANDRA* spectra for 9 clusters from the samples of Vikhlinin et al. (2006; 2009)

Used same spectra, cosmology, and values for $r_{2500}$ and $r_{500}$

Note: $r_{j+1} = 1.5 r_j$ for $j > 1$ (larger than ideal)

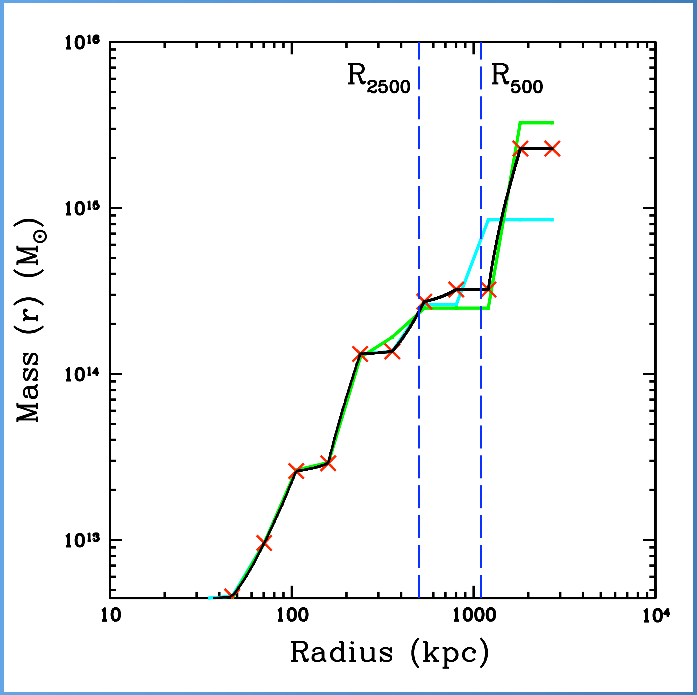
| Cluster | z | $r_{2500}$ [a] (kpc) | $r_{500}$ [a] (kpc) | *Chandra* ObsID |
|---|---|---|---|---|
| A907 | 0.1603 | 501 | 1095 | 3185, 3205, 535 |
| A1413 | 0.1429 | 559 | 1300 | 1661, 5002, 5003 |
| A1991 | 0.0592 | 341 | 734 | 3193 |
| A2029 | 0.0779 | 642 | 1359 | 891, 4977, 6101 |
| A2390 | 0.2302 | 561 | 1414 | 4193 |
| A1835 | 0.2520 | 673 | 1475 | 6880, 6881, 7370 |
| A1650 | 0.0845 | 515 | 1128 | 5822, 5823, 6356, 6357, 6358, 7242 |
| A3112 | 0.0761 | 459 | 1025 | 2216, 2516, 6972, 7323, 7324 |
| A2107 | 0.0418 | 416 | 919[b] | 4960 |

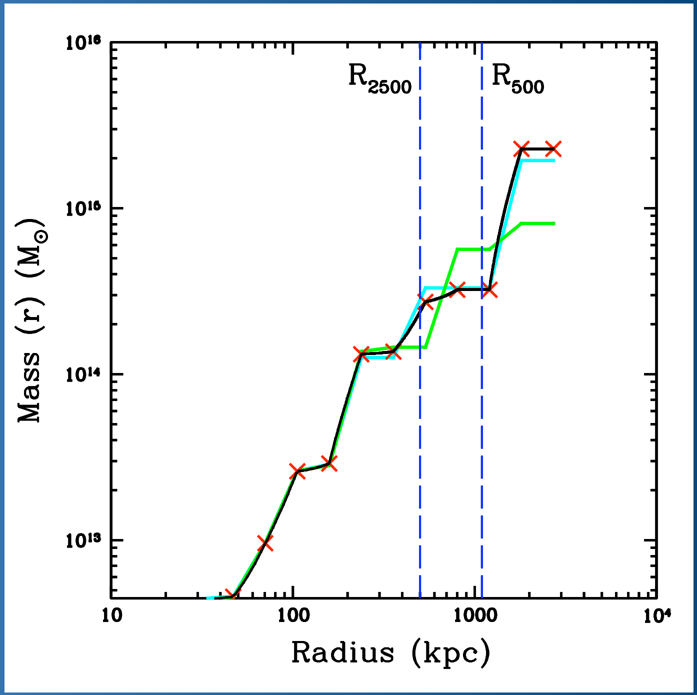
# Mass Profiles for Abell 907

Gravitating mass, $M(r) = \sum_i \rho_i V_i(< r)$ where $V_i(<r)$ is the volume of shell $i$ lying inside $r$

For fixed $M$, this provides a constraint on one $\rho_k$, so that confidence ranges for $M$ can be determined within XSPEC

Best fit mass profile (black), with upper and lower 90% confidence limits at $R_{500}$ (left) and $R_{2500}$ (right) – *mass densities for individual shells are not well constrained by CLMASS*



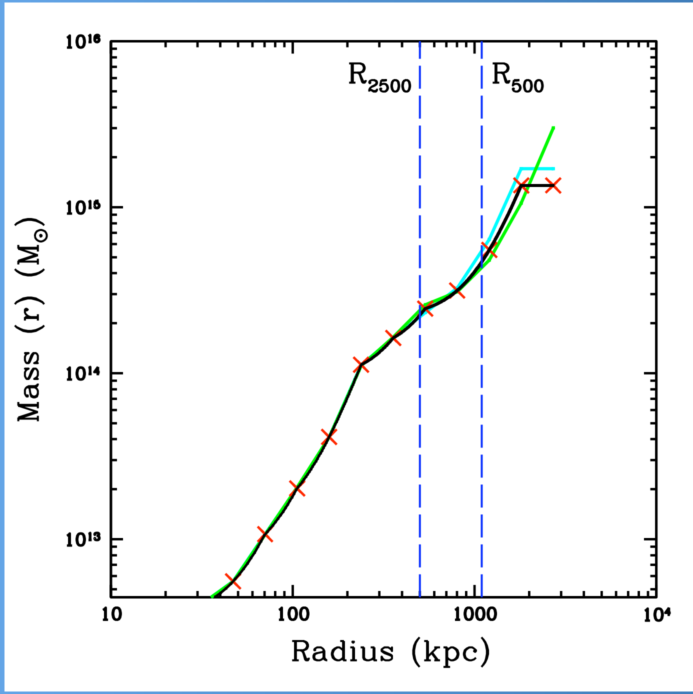



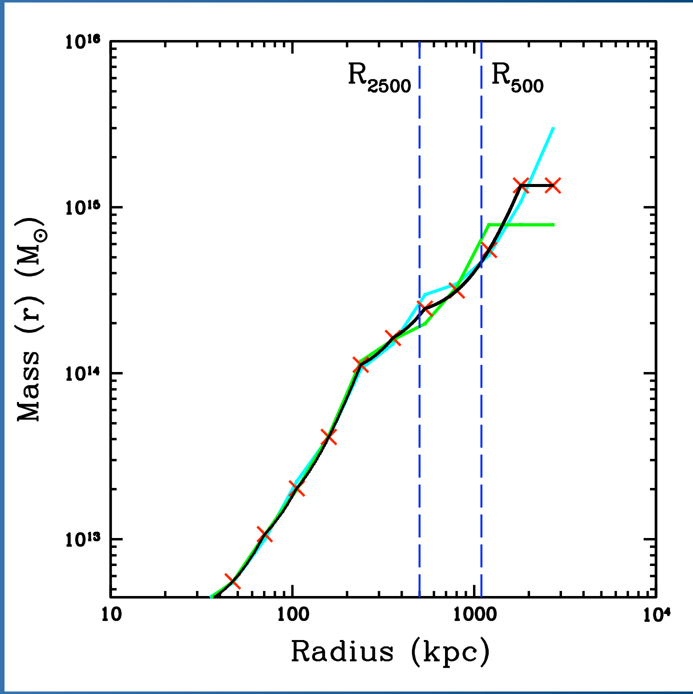
# Mass Profiles for Abell 907

Constraining the gravitating mass density to be a monotonically decreasing function of the radius ($\rho_i \geq \rho_{i+1}$) gives better results

Best fit montonic mass profile (black), with upper and lower 90% confidence limits at $R_{500}$ (left) and $R_{2500}$ (right)

Prefer monotonic mass profiles



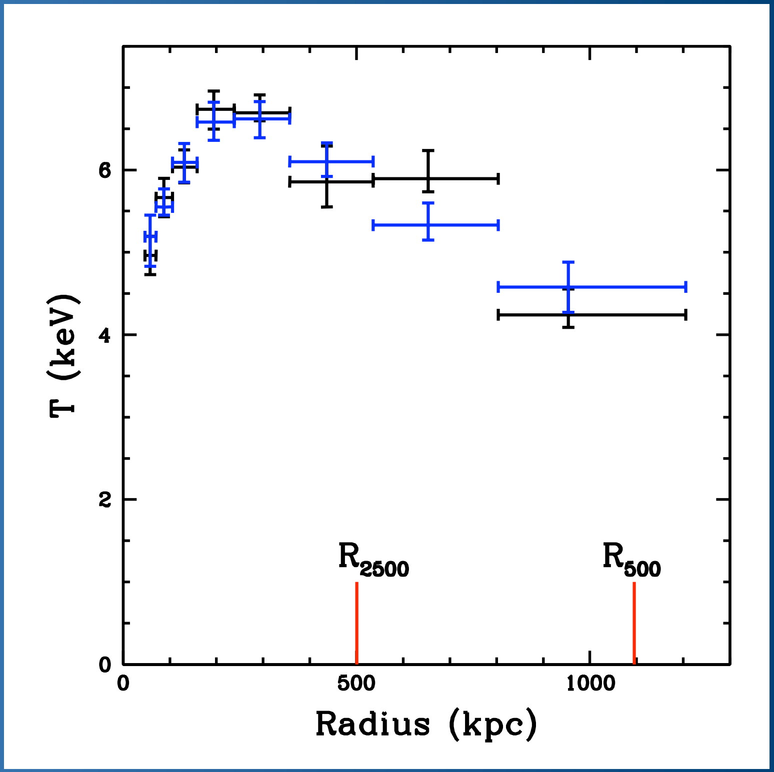



# Temperatures for Abell 907

Black points show temperatures for the best fitting monotonic CLMASS model (1σ error bars)

Blue points show (projected) shell temperatures from Vikhlinin et al (2005) (with same spectra)

Temperature deprojection in CLMASS model makes temperatures noisier



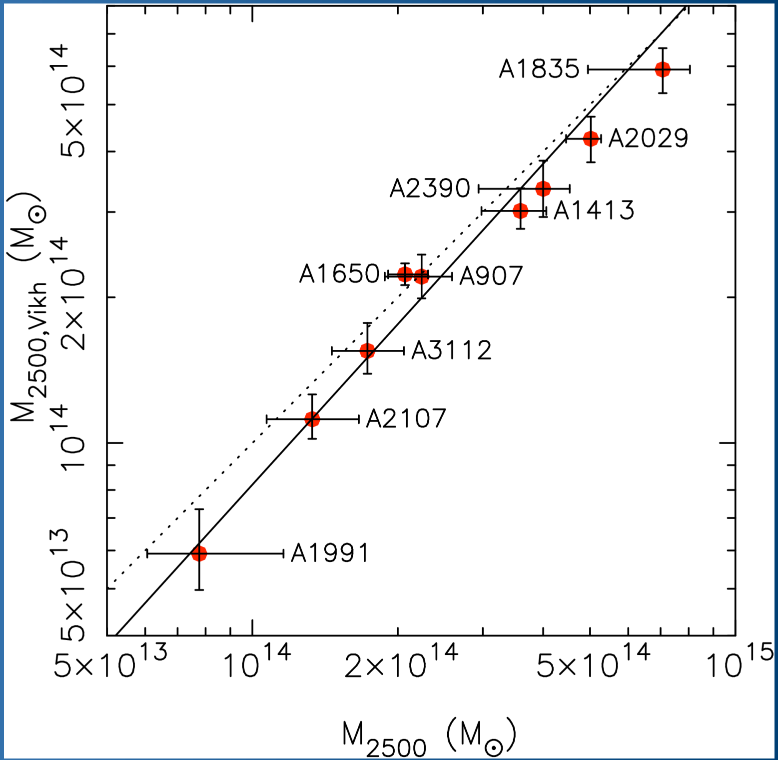
# Comparison to $M_{2500}$ from Vikhlinin et al

90% confidence ranges for $M_{2500}$ from monotonic CLMASS model compared to masses from Vikhlinin et al (2005)

Average 90% spread of 45% for CLMASS is comparable to 25% spread for model-dependent results

Best fit line (by bisector method of Akritas & Bershady 1996) is

$$\log_{10}\frac{M_{2500,\mathrm{Vikh}}}{10^{14}\,\mathrm{M_{sun}}} = -0.086 + 1.101\log_{10}\frac{M_{2500}}{10^{14}\,\mathrm{M_{sun}}}$$




Monte Carlo simulations show that the slope is within 1σ of unity, but CLMASS results are 12% high at the mean, which only occurs 2% of the time

Moderate offset probably due to the wide annuli used here ($r_{j+1} = 1.5 r_j$)

# Mixing Models in Sherpa

A general mixing model uses a weighted sum of several input models to produce multiple output models – e.g., PROJCT, CLMASS, but also models correcting for PSF, etc.

XSPEC mixing models are limited – e.g., they must be last (no extra BG model, or variable foreground absorption).

Sherpa is far more flexible: example mixing model in sherpa (for annulus number 4)
0.916 * (xswabs.wa4 * (annulus4 * mix * shell1 * xsmekal.mk1 + annulus4 * mix * shell2 * xsmekal.mk2 + annulus4 * mix * shell3 * xsmekal.mk3 + …))

mix is an object of class MixMod that computes and keeps the mixing matrix
annulus1, shell1, etc. belong to class MixModProj – they simply keep track of a component number
Left and right multiplication are overloaded for class MixMod, so that annulus4 * mix * shell3 evaluates to the (4, 3) component of the matrix kept by mix.

Approach can be used for any mixing model.

# Conclusions

- Gas densities, temperatures and abundances can be determined from X-ray spectra for clusters
- Masses of spherical, hydrostatic galaxies, groups and clusters can also be determined from high quality X-ray data without model-dependent assumptions
- XSPEC and Sherpa mixing models exist to perform this task
- High quality *CHANDRA* data still do not constrain cluster masses at large R well
- Generic mixing models can be implemented efficiently by taking advantage of the python interface of Sherpa

# Example: Fitting a Navarro Frenk & White Potential

The tar file includes 10 annular spectra (for Abell 2029), with XFLT keywords set for use in the XSPEC NFWMASS model, the local model code for three mass models, supporting TCL scripts and C++ programs.

Note that the method used in the example is NOT model-independent.

You can use this as an example of:

- building local models in XSPEC
- fitting models with many parameters
- scripting fitting tasks
- writing your own local model

# Comparison to $M_{500}$ from Vikhlinin et al

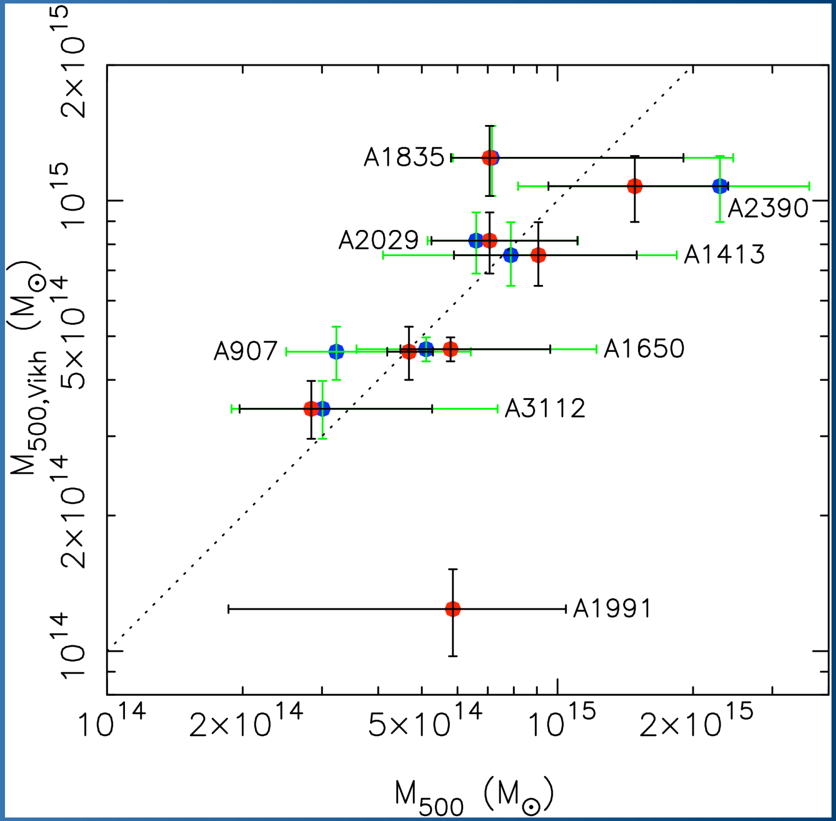
90% confidence ranges for $M_{500}$ from CLMASS compared to the same from Vikhlinin et al (2005).

Green error bars and blue points show unconstrained results (average length 3.5× in mass)

Black error bars and red points show monotonic results (average length 2.3× in mass)

CHANDRA data with the CLMASS model only provides loose constraints on $M_{500}$

To obtain tighter mass limits, data must extend well beyond radius of interest



# Strengths and Weaknesses

The bare CLMASS model constrains the gravitating matter densities of individual shells poorly

– does better if the matter densities are assumed to decrease monotonically with R (model MONOMASS)

Without model-dependent assumptions, existing X-ray data only constrain cluster masses weakly at large R

– get much better constraints deeper within clusters (at $R_{2500}$)

Model-dependent assumptions are an issue for all mass determinations

Use of the CLMASS model needs to be optimized

– e.g. tradeoffs in shell width – narrower shells would exploit surface brightness information better, but many photons are needed for good spectra

Physical model can be used for any parametrization of the potential, e.g., NFWMASS

# CLMASS Results

| Cluster | Unconstrained $\chi^2/\mathrm{dof}$ | Unconstrained $M_{500}$ ($10^{14}\ M_\odot$) | Monotonic $\chi^2/\mathrm{dof}$ | Monotonic $M_{500}$ ($10^{14}\ M_\odot$) | Monotonic $M_{2500}$ ($10^{14}\ M_\odot$) |
|---|---|---|---|---|---|
| A907 | 692.4/626 | $3.2^{+3.2}_{-0.7}$ | 693.7/630 | $4.7^{+0.6}_{-0.5}$ | $2.2^{+0.4}_{-0.3}$ |
| A2390 | 840.9/626 | $23.0^{+13.3}_{-14.8}$ | 846.2/632 | $14.9^{+9.1}_{-5.3}$ | $4.0^{+0.5}_{-1.1}$ |
| A1835 | 489.4/290 | $7.1^{+17.4}_{-1.3}$ | 491.7/293 | $7.1^{+12.0}_{-1.3}$ | $7.1^{+1.0}_{-2.1}$ |
| A1650 | 669.1/301 | $5.1^{+7.1}_{-1.5}$ | 673.2/306 | $5.8^{+3.9}_{-1.3}$ | $2.1^{+0.2}_{-0.2}$ |
| A3112 | 545.8/314 | $3.0^{+4.4}_{-1.1}$ | 546.7/317 | $2.8^{+2.4}_{-0.9}$ | $1.7^{+0.3}_{-0.3}$ |
| A2029 | 2988.0/664 | $6.6^{+4.5}_{-1.5}$ | 2993.5/669 | $7.1^{+4.0}_{-1.8}$ | $5.0^{+0.3}_{-0.6}$ |
| A1991 | | | 563.7/326 | $5.9^{+4.6}_{-4.0}$ | $0.8^{+0.4}_{-0.2}$ |
| A1413 | 449.0/313 | $7.9^{+10.5}_{-3.9}$ | 450.3/321 | $9.1^{+5.9}_{-3.2}$ | $3.6^{+0.5}_{-0.6}$ |
| A2107 | | | 428.1/299 | | $1.3^{+0.3}_{-0.3}$ |

90% confidence ranges

No results at $R_{500}$ for Abell 2107, since it is not reached by *CHANDRA*

No unconstrained result at $R_{500}$ for Abell 1991 because CLMASS gave no useful 90% upper limit