Careful with the priors:
a reanalysis of LIGO black-hole coalescences

Davide Gerosa

NASA Einstein Fellow

California Institute of Technology

arXiv:1707.04637

with S. Vitale, C.-J. Haster, K. Chatziioannou, A. Zimmerman

Caltech

October 13th, 2017

Einstein Fellow Symposium

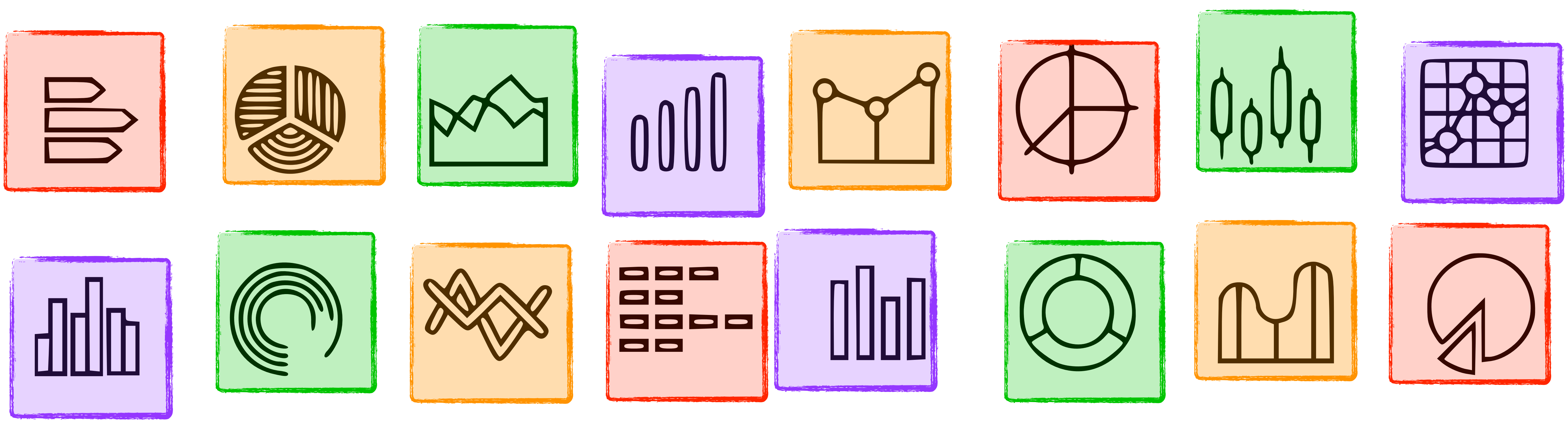
Cambridge MA

dgerosa@caltech.edu

www.tapir.caltech.edu/~dgerosa

Outline

1. Gravitational waves and priors
2. A reanalysis of current black-hole detections
3. Results: prior matters (sometimes)



On the shoulders of giants

There is reason to expect an event with more or less confidence according to the greater or less number of times in which, under given circumstances, it has happened without failing

Thomas Bayes, 1763

On the shoulders of giants

Likelihood: how probable is the data given some parameters?

Prior: how probable is a set of parameters before getting new data?

$$\boxed{P(\theta|d)} = \frac{\boxed{P(d|\theta)}\ \boxed{p(\theta)}}{\boxed{\int P(d|\theta)p(\theta)d\theta}}$$

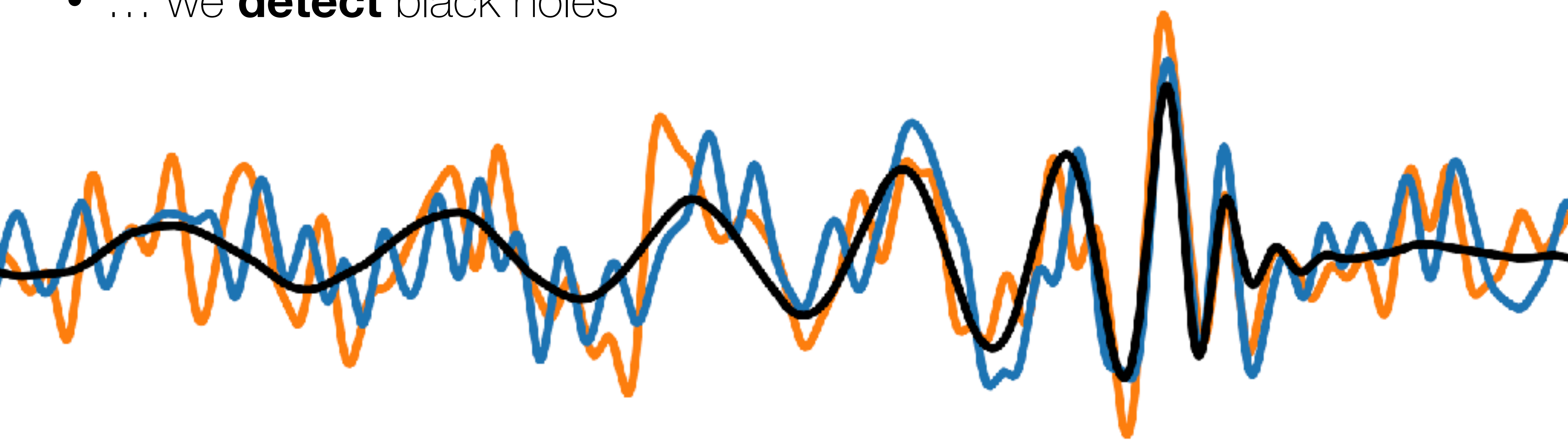
Posterior: how probable are these parameters given the observed data?

Evidence: how probable is the data under all possible parameters?

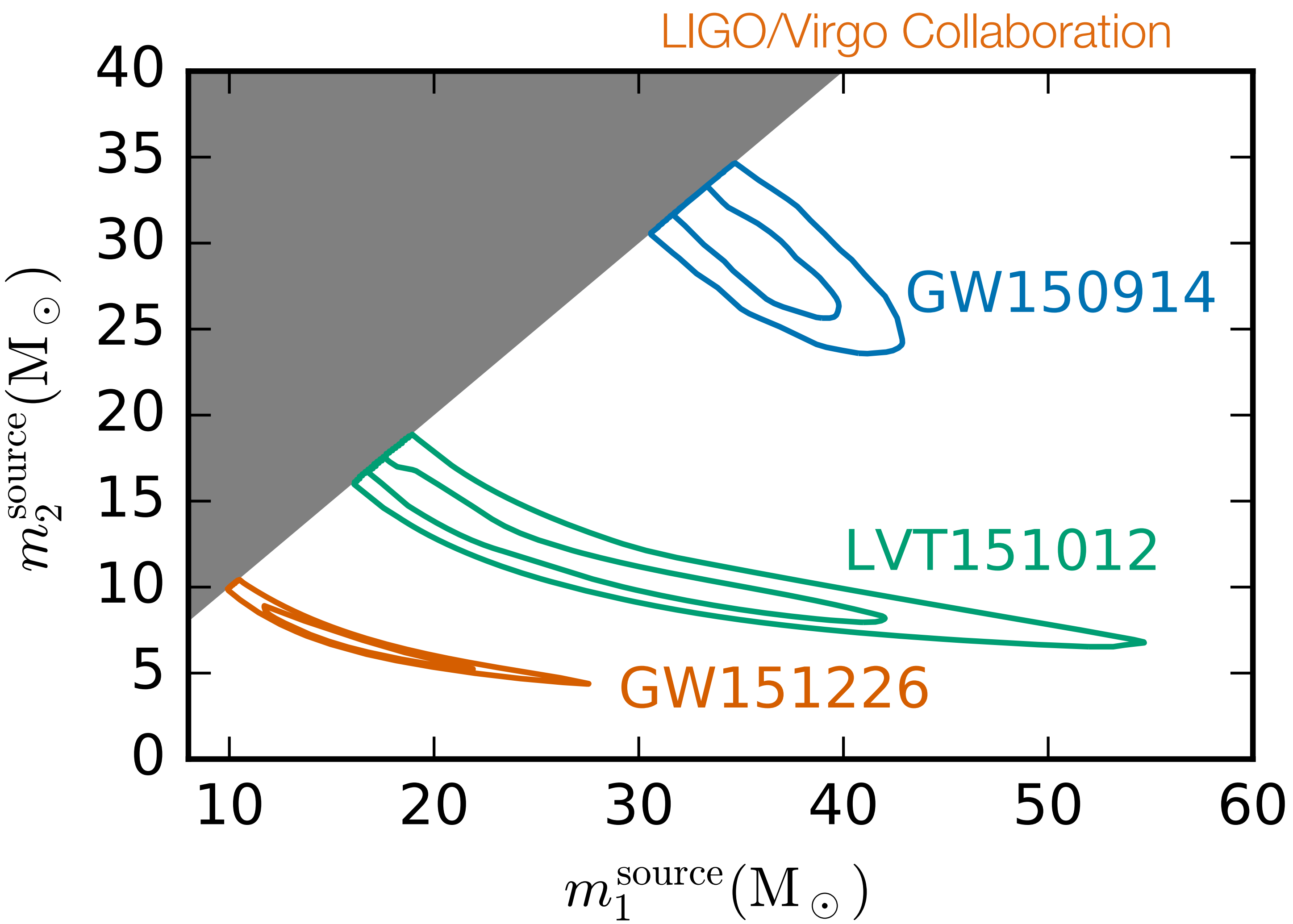
This is really the scientific method: one always approaches a problem with an hypothesis on it.
How does data update my understanding?

GWs: a gigantic set of priors!

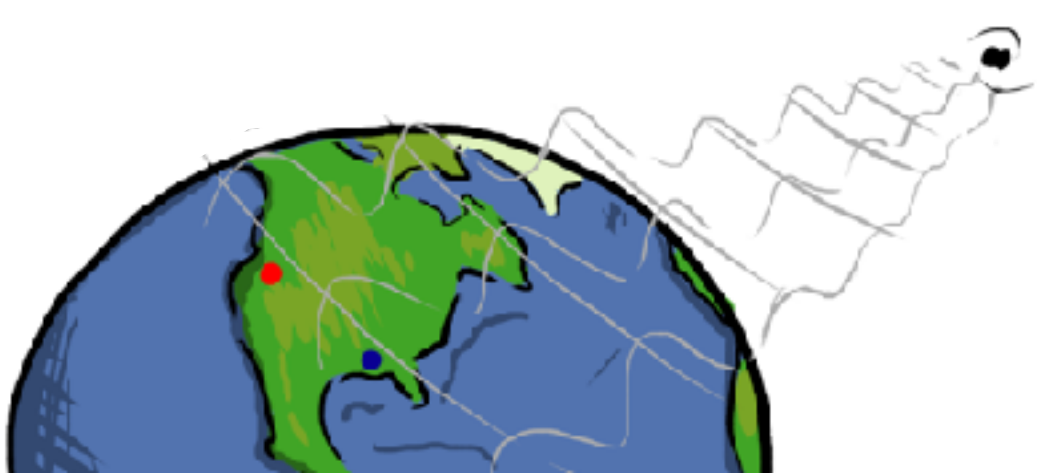
- Gravitational waves are predicted **by GR.**
- **GR has passed all tests with flying colors.** We have a huge preconception that GR is an accurate description of reality.
- Indeed, we talk about detecting a deviation **from GR** not about measuring the theory of gravity.
- GR **predicts** black holes…
- … we **detect** black holes



Mass measurements



- Low mass: many orbits; chirp mass: $$M_c = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$$
- High mass: mainly merger; total mass: $$M_{\rm tot} = m_1 + m_2$$
- Data analyzed with priors **uniform in component masses** m_1, m_2



This is a specific assumption we're inserting into the analysis

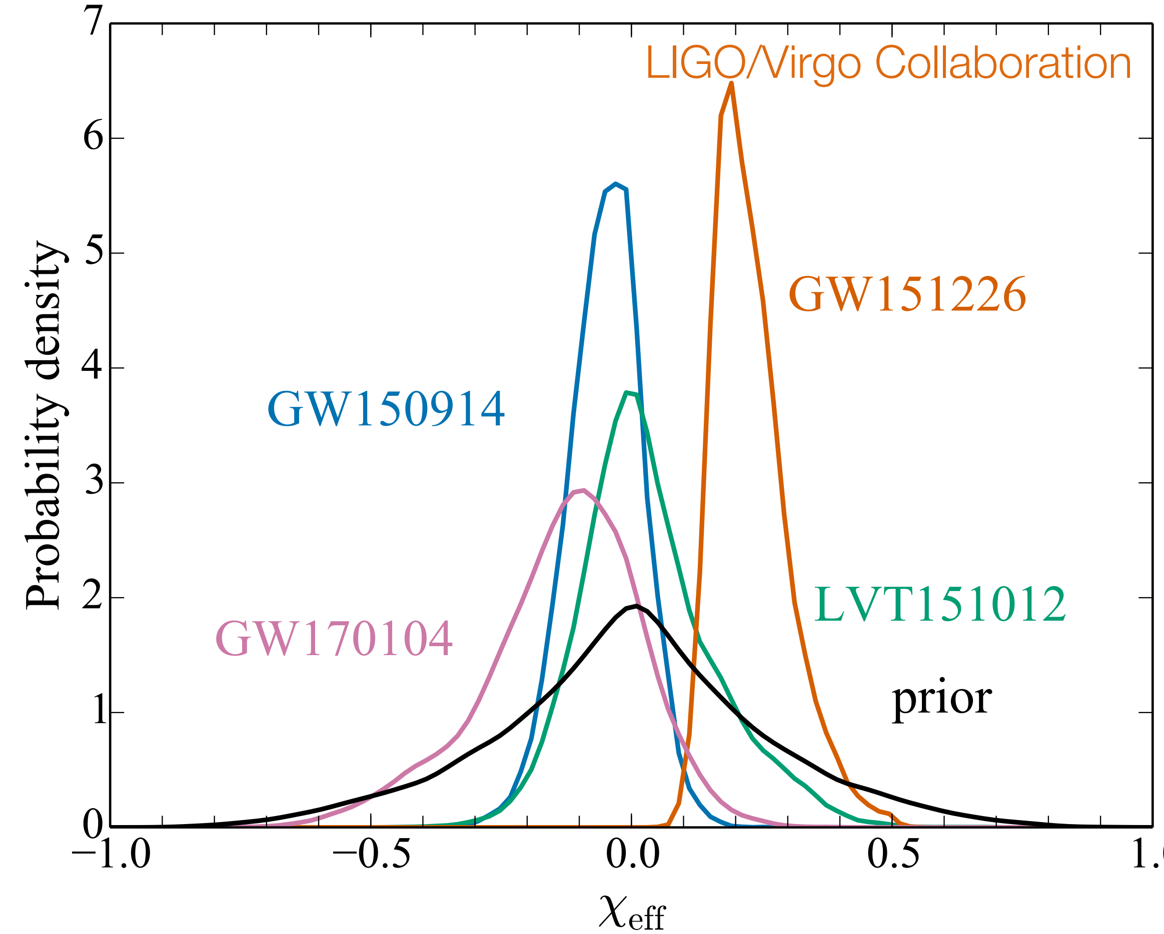
Spin measurements

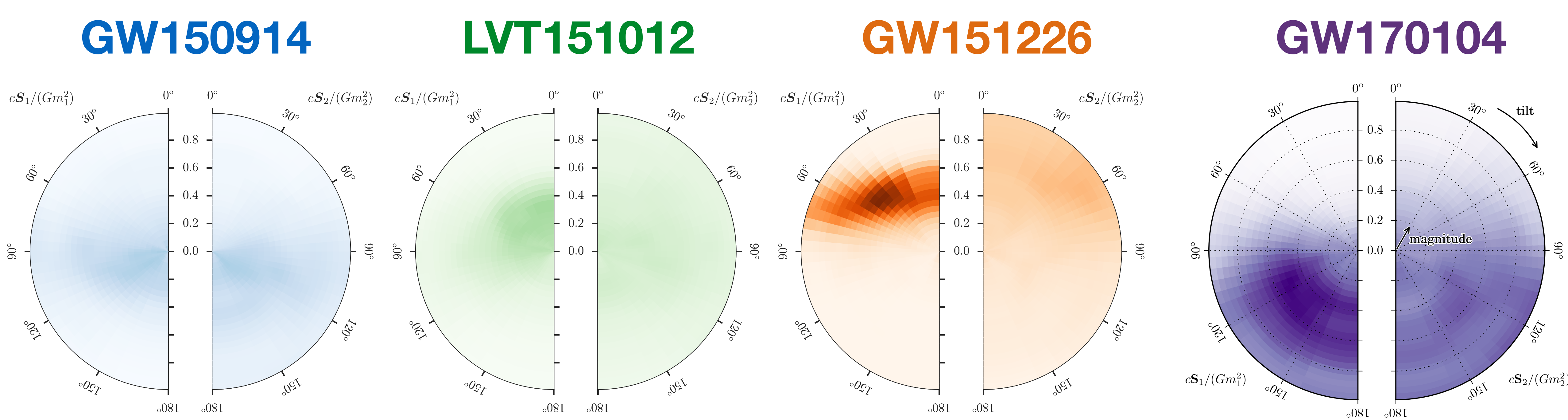
- Best measured quantity: effective spin

$$\chi_{\rm eff} = \left(\frac{\mathbf{S_1}}{m_1} + \frac{\mathbf{S_2}}{m_2} \right) \frac{\hat{\mathbf{L}}}{M}$$

- Constant of motion at 2PN Racine 2008; **DG**+ 2015
- Priors **uniform in spin magnitude** and **isotropic in spin direction** at 20Hz!

This is a specific assumption we're inserting into the analysis





Does it matter?

$$\boxed{P(\theta|d)} = \frac{\boxed{P(d|\theta)}\ \boxed{p(\theta)}}{\boxed{\int P(d|\theta)p(\theta)d\theta}}$$

If data are strong:

$$\boxed{P(\theta|d)} = \frac{\boxed{P(d|\theta)}\ p(\theta)}{\int P(d|\theta)p(\theta)d\theta}$$

If data are weakly informative:

$$\boxed{P(\theta|d)} = \frac{P(d|\theta)\ \boxed{p(\theta)}}{\int P(d|\theta)p(\theta)d\theta}$$

How informative are current GW data?
Which events? Which parameters?

What prior knowledge could go into a black hole analysis?

Black holes have spins

- Spins are vectors, magnitude and direction.
- Rotating bodies have rotational energy $E_{\rm rot} \equiv 1 - \sqrt{1+\sqrt{1-\chi^2}}/\sqrt{2}$

Black holes come from stars

- Masses of stars are not all equally probable Kroupa 2001, Bastian+ 2010
- Black hole spins from stellar collapse might be low Spruit 2002, Fuller+ 2015
- But X-ray binary measurements suggest spins are high. Bimodal? Miller & Miller 2015
- Stellar interactions might align the BH spins… Hut 1981, Belczynski+ 2008, **DG**+ 2013
- … but dynamical interactions predict isotropic spins **This is the current prior!**

Let's give it a try

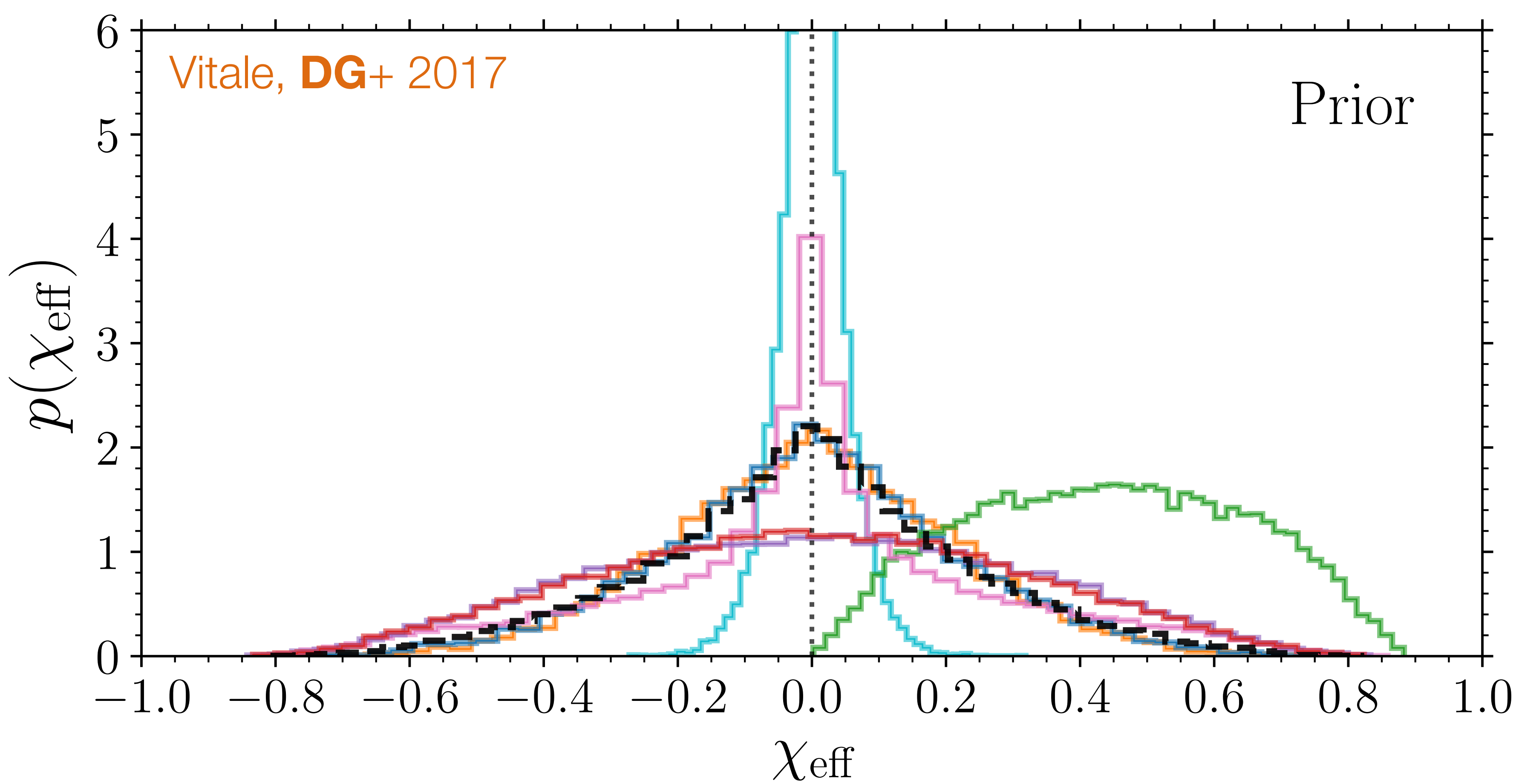
- P_1 Default: everything is uniform and isotropic
- P_2 Spins uniform in BH rotational energy
- P_3 Spins uniform in volume
- P_4 Bimodal in the spin magnitudes
- P_5 Spins preferentially aligned
- P_6 Stellar initial mass function
- P_7 Stellar initial mass function v2
- P_8 Small spin magnitudes

	Individual masses	Spin magnitude	Spin Direction
P_1	Uniform	Uniform	Isotropic
P_2	Uniform	Uniform in $E_{\rm rot}$	Isotropic
P_3	Uniform	Volumetric	Isotropic
P_4	Uniform	$\mathcal{N}(0,0.1)+\mathcal{N}(0.89,0.1)$	Isotropic
P_5	Uniform	Uniform	$\mathcal{N}(0,10°)$
P_6	Power law	Uniform	Isotropic
P_7	Logistic	Uniform	Isotropic
P_8	Uniform	$\mathcal{N}(0,0.1)$	Isotropic

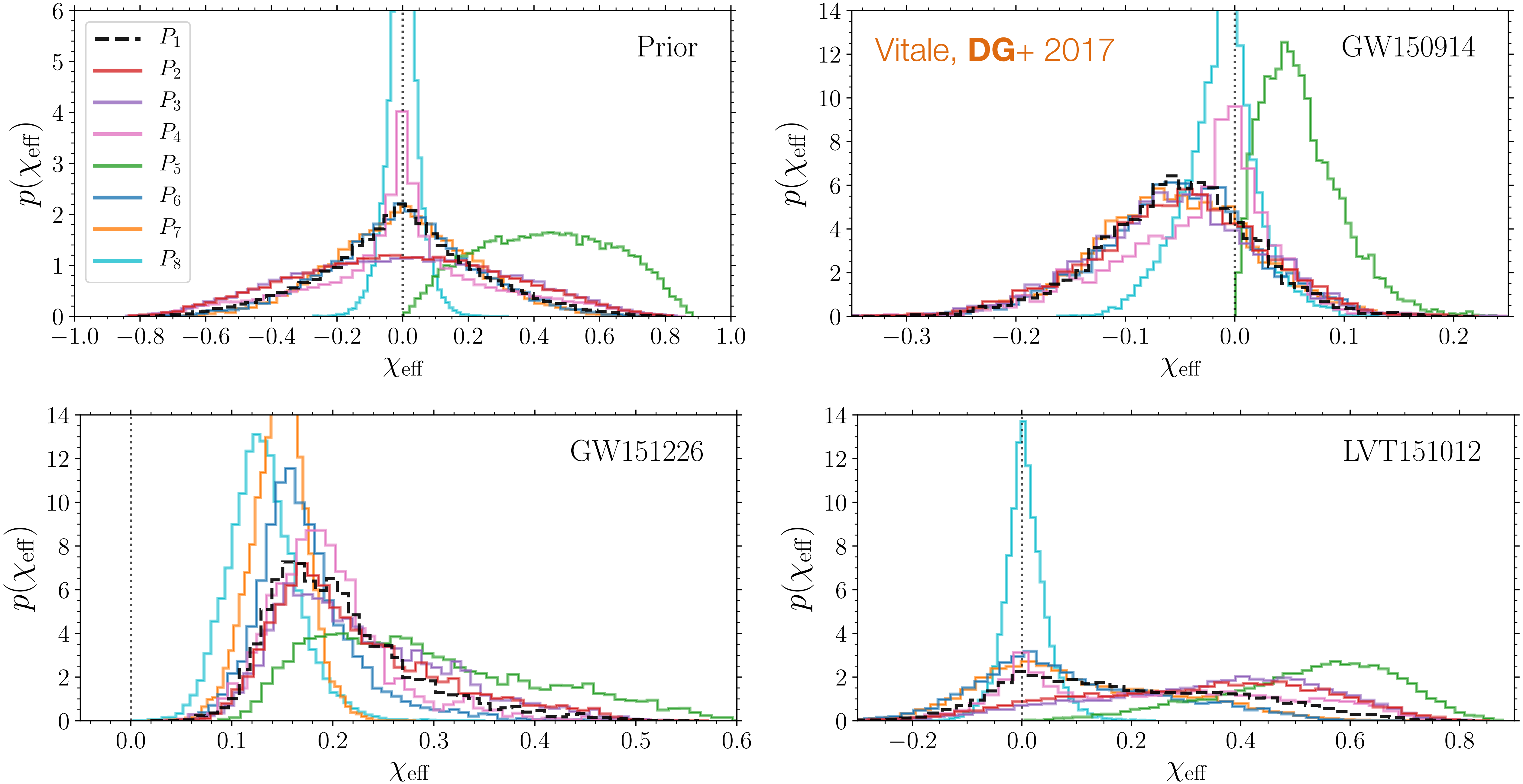
Note: this is *the very first independent reanalysis of the LIGO data.*

Equally good

	GW150914 $\log_{10}\mathcal{O}$	GW151226 $\log_{10}\mathcal{O}$	LVT151012 $\log_{10}\mathcal{O}$
P_1	—	—	—
P_2	-0.3	0.0	-0.1
P_3	-0.4	0.0	0.0
P_4	0.0	-0.1	-0.1
P_5	-1.7	0.0	0.5
P_6	0.1	0.4	0.4
P_7	0.4	0.4	0.5
P_8	0.3	-1.7	-0.1



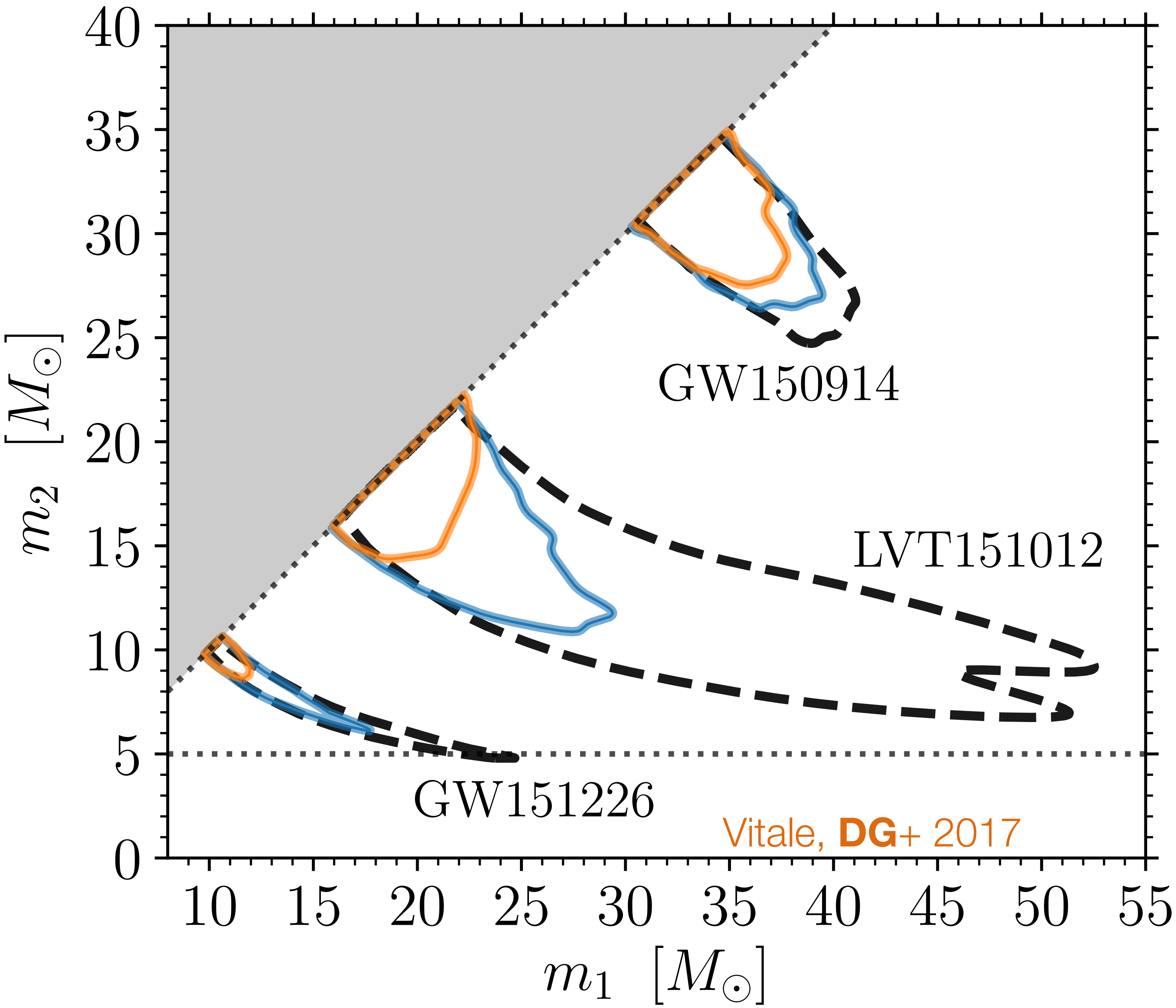
Impact on inferred BH spins



- GW151226 not consistent with zero spins (robust!)
- The bimodal spin prior choses the high spin mode. Support misalignment.
- All others fully consistent with zero spins (robust!)
- More severe issues for low SNR like LVT

Variations in the 90% confidence interval up to ~20%!

Impact on inferred BH masses



- Chirp mass (GW151226 and LVT151012), total mass (GW150914) are **very solid**.
- Median change of $\sim 0.1 M_\odot$
- But component masses are not

If you insert the analysis the information that BH should come from stars:…

- **Data tends to favor more equal mass systems**
- **…especially if info from dynamical interactions are in**

P_1	Default: everything is uniform and isotropic
P_6	Stellar IMF, uniform mass ratio Sana+ 2012
P_7	Stellar IMF, logistic mass ratio Rodriguez+ 2016

Is there a mass gap between BHs and NSs?
Miller & Miller 2015; Kreidberg 2012

Astro models should be incorporated as priors to obtain data constraints, then model selection

Was it necessary?

One set of assumptions:

$$P(\theta|d) = \frac{P(d|\theta)\, p(\theta)}{\int P(d|\theta) p(\theta) d\theta}$$

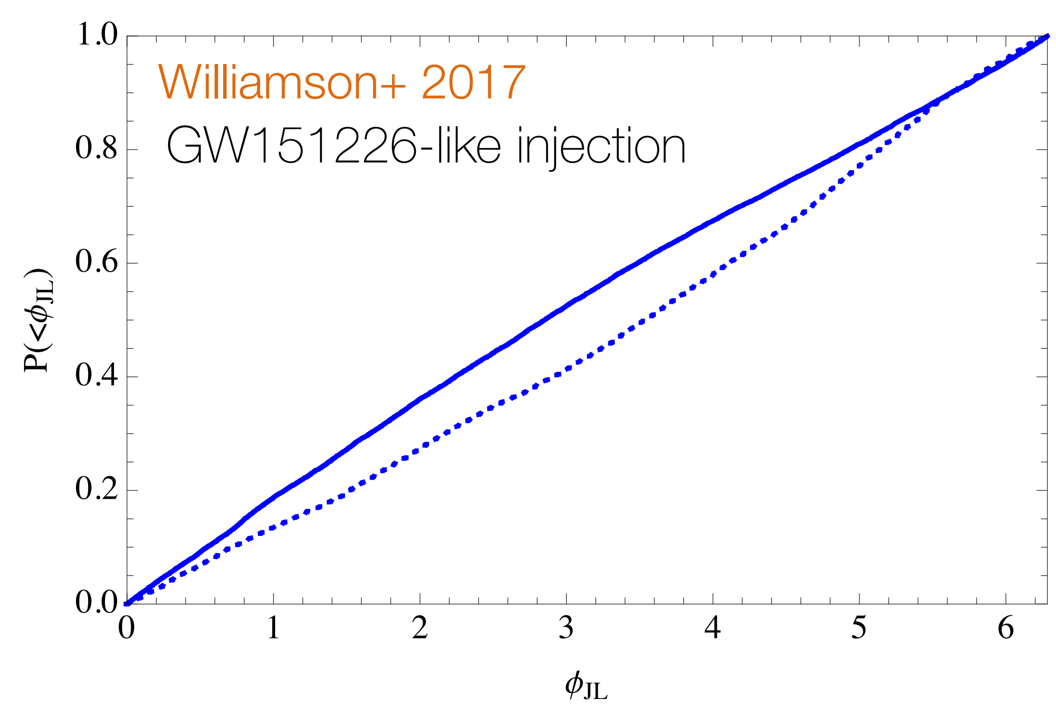
Another one:

$$\tilde{P}(\theta|d) = \frac{P(d|\theta)\, \tilde{p}(\theta)}{\int P(d|\theta) \tilde{p}(\theta) d\theta}$$

$$\tilde{P}(\theta|d) \propto P(\theta|d) \frac{\tilde{p}(\theta)}{p(\theta)}$$

Can one just reweigh the posterior to access the likelihood?

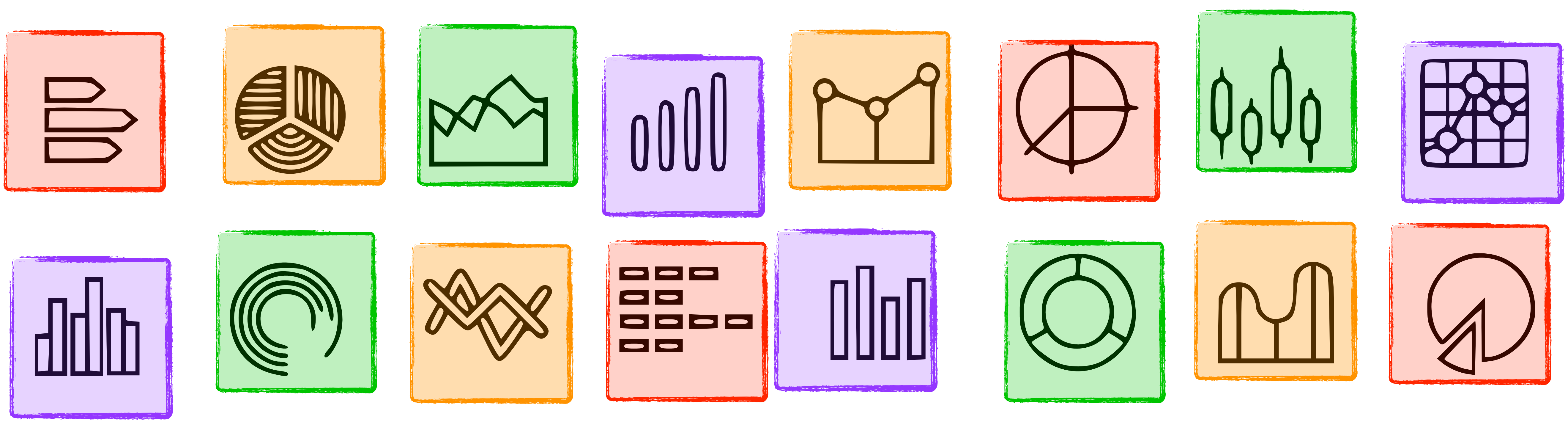
- Used heavily in hierarchical model selection, to combine more observations
- Can be done, but might be dangerous
- Systematics must be treated carefully



Outline

arXiv:1707.04637

1. Gravitational waves and priors
2. A reanalysis of current black-hole detections
3. Results: prior matters (sometimes)



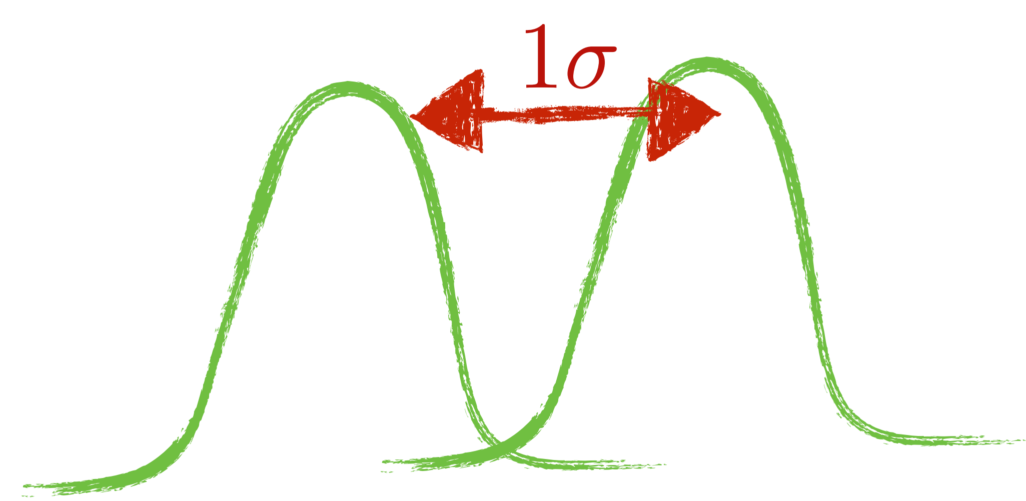
Backup slides

Kullback-Leibler divergence

$$\boxed{P(\theta|d)} = \frac{\boxed{P(d|\theta)}\,\boxed{p(\theta)}}{\boxed{\int P(d|\theta)p(\theta)d\theta}}$$

$$D_{\mathrm{KL}} = \int d\theta\, \boxed{P(d|\theta)}\, \ln\left(\frac{\boxed{P(d|\theta)}}{\boxed{p(\theta)}}\right)$$

	GW150914			GW151226			LVT151012		
	$D_{\mathrm{KL}}^{\chi_{\mathrm{eff}}}$	$D_{\mathrm{KL}}^{\chi_{\mathrm{p}}}$	$\log_{10}\mathcal{O}$	$D_{\mathrm{KL}}^{\chi_{\mathrm{eff}}}$	$D_{\mathrm{KL}}^{\chi_{\mathrm{p}}}$	$\log_{10}\mathcal{O}$	$D_{\mathrm{KL}}^{\chi_{\mathrm{eff}}}$	$D_{\mathrm{KL}}^{\chi_{\mathrm{p}}}$	$\log_{10}\mathcal{O}$
P_1	1.02	0.03	—	1.93	0.21	—	0.53	0.03	—
P_2	1.36	0.06	-0.3	1.78	0.04	0.0	0.89	0.05	-0.1
P_3	1.52	0.09	-0.4	1.76	0.02	0.0	0.95	0.04	0.0
P_4	0.88	0.12	0.0	2.56	0.70	-0.1	0.61	0.12	-0.1
P_5	4.21	1.75	-1.7	0.82	0.21	0.0	0.22	0.07	0.5
P_6	0.96	0.01	0.1	2.12	0.08	0.4	0.24	0.00	0.4
P_7	0.93	0.06	0.4	2.63	0.02	0.4	0.26	0.01	0.5
P_8	0.14	0.07	0.3	4.82	0.70	-1.7	0.03	0.02	-0.1



$D_{\mathrm{KL}} = 0.72$ bit