Detecting Diffractive Lensing in Astrophysical Gravitational Waves

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Gravitational Waves: A New Window Into the Universe

Masses in the Stellar Graveyard

- LIGO-Virgo Black Holes
- X-ray Binary Black Holes
- Known Neutron Stars
- LIGO-Virgo Neutron Stars

Space-based observatory (LISA)
Third generation detector

LIGO Hanford/Livingston
Virgo
KAGRA
LIGO-India
Gravitational lensing of gravitational waves

- Regime of geometrical lensing
  \[ \lambda_{GW} \ll R_{de} \]
- Amplification of strain
  \[ h' = \sqrt{\mu} \, h \]
- Wave frequency unchanged

compact binary coalescence
Intervening mass clump
GW waveform
detector

Wang, Stebbins & Turner 96’
Li, Mao, Zhao & Lu 18’
Observational difficulty with geometrical lensing

Dai, Venumadhav & Sigurdson 2017
Ng, Wang, Broadhurst & Li 2017
Broadhurst, Diego & Smoot 2018
Oguri 2018

Apparent mass scale $M'$
Apparent source redshift $z'$

\[ M (1 + z) = M' (1 + z') \]
\[ \frac{\sqrt{\mu}}{d_L(z)} = \frac{1}{d_L(z')} \]

Without EM observations, magnification cannot be recovered from a single lensed image

Topological (Morse) phase shift for flipped images; however degenerate with orbital orientation; Dai & Venumadhav 2017

Multiple lensed images resolvable in time domain
Require fine-tuned impact parameters and large column density
Can be produced by cluster/galaxy lenses; difficult for low-mass lenses
Lensing in wave-diffraction regime

\[ h(f) = F(f) h_0(f) \]

amplification factor

Limit of geometric optics

\[ \omega = 2\pi f (1 + z_L) G M_L/c^2 \gg 1 \]

(if only one image)

\[ F_{geo}(f) \rightarrow \sqrt{\mu_{geo}} e^{i 2\pi f (1+z_L) \tau_{geo}} \]

Regime of wave diffraction

\[ \omega = 2\pi f (1 + z_L) G M_L/c^2 \sim \mathcal{O}(1) \]

Define

\[ F_{rel}(f) := \frac{F(f)}{F_{geo}(f)} \]

Encode information of waveform distortion!
Amplitude and Phase Modulations

Ground-based band $f \sim 10$-1000 Hz sensitive to (interestingly) small lens masses $M \sim 100$-1000 Msun  Dai, Li, Zackay, Mao & Lu 2018

Pseudo-Jaffe lens
Impact parameter $\sigma_v \sim 1 - 2$ km/s
$\theta_E \sim 4\pi (\sigma_v/c)^2$

Size of modulation inversely proportional to the impact parameter
Lens mass scale increases
Diffraction signature is subtle

Small modulus and phase perturbations ~ 10-20% or even smaller!

Can you see the amplitude/phase modulations?
Diffraction signature is subtle

Small modulus and phase perturbations $\sim 10$-$20\%$ or even smaller!

Can you see the amplitude/phase modulations? Hmm ... not so impressive ...
Match with unlensed templates is (nearly) unaffected.

Diffraction signature still detectable through the improvement in the likelihood when amplitude/phase modulations are included into the waveform.

\[
\text{match} := \frac{\langle h_L | h_{BF} \rangle}{\sqrt{\langle h_L | h_L \rangle \langle h_{BF} | h_{BF} \rangle}}
\]

\[
\ln p = - \frac{1}{2} \left( \langle h_L | h_L \rangle - \frac{\langle h_L | h_{BF} \rangle^2}{\langle h_{BF} | h_{BF} \rangle} \right)
\]
Matched filtering and some practical difficulties

- Matched filtering requires the precise knowledge of $F(f)$
  
  e.g. Takahashi & Nakamura 2003, Cao+ 2014, Jung & Shin 2017

- $F(f)$ depends on too many parameters: lens profile, distances, impact parameter, etc.

- The correct lens profile to use is unknown.

- Have to search with a large number of templates. Look-elsewhere effect needs to be quantified.
An agnostic method based on dynamic programming

\[ S := \int \mathcal{D}g(f) \mathcal{P}[g(f)] \prod_{a=1}^{N_d} \frac{P[s_a(f)|g(f) h_{BF,a}(f)]}{P[s_a(f)|h_{BF,a}(f)]} \]

prior (Markovian)

likelihood improvement

\[ h_{BF}(f) \text{ is the best-fit unlensed waveform} \]

\[ F_{rel}(f) - 1 \]

detector noise

diffraction distortion

Dai+ 2018
Reconstructing the modulations

$M_c = 1.2 \, M_\odot$, $\eta = 0.24$, $\Lambda_{1,2} = 400$, $s_{1z} = s_{2z} = 0$

$S = 7.783$

Pseudo-Jaffe lens at $z = 0.1$

Velocity dispersion $\sigma_v = 2 \, \text{km/s}$

NS-NS merger at $z = 0.2$

Can only detect the part of the modulation signal that is not degenerate with source parameters!
Observational prospect: BH mergers are promising!

\[ M_c = 30 \, M_\odot, \quad \eta = 0.24, \quad s_{1z} = s_{2z} = 0 \]

- aLIGO (×2)
  - \( z_s = 0.24 (1.2) \)
  - \( \sigma_v = 2 \text{ km/s} \)

- ET
  - \( z_s = 2 (15) \)
  - \( \sigma_v = 1.5 \text{ km/s} \)

Assume pseudo-Jaffe halos; mass enclosed within the Einstein radius \( \sim 100 \text{ --- } 1000 \text{ Msun} \)

- aLIGO Hanford/Livingston can probe out to \( z \sim 0.2 - 0.3 \).
- Further improves after more detectors join (Virgo, KAGRA, LIGO-India, etc)
- 3\textsuperscript{rd} generation detector will be very powerful: \( z \sim 2 - 4 \)
Science case: test CDM theory on sub-galactic scales ?!

- Probe inner region of $M \sim 10^4$—$10^6$ Msun DM halos
- 3rd gen. detector can use BBHs out to $z \sim 2$-4
- Assume nearly log-flat halo mass function; lensing optical depth $\sim$ few $\times 10^{-3}$ if $r_E \sim 1$ pc
- Enough enclosed mass? Small halos show steeper inner profiles than NFW.
  
  e.g. Dutton & Maccio 2014
- Galaxy lensing events particularly interesting to look at
Thank you!
Degeneracy with **spin-precessing** or **eccentricity** effects?

- Precession can induce amplitude/phase modulations in the frequency-domain waveform. *e.g. Apostolatos, Cutler, Sussman & Thorne, 1994; Klein, Cornish & Yunes 2013*

- Modulation frequency (~ tens of precession cycles in band) is typically higher than diffraction; amplitude modulation more significant than phase modulation.

- Detailed study would be very valuable; need accurate waveforms.