Coordinate Systems for Analysis of On-Orbit Chandra Data
Paper II: Aspect Calibration

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March 9, 2000

1 Introduction

The Chandra X-ray Observatory, launched in July 1999, provides X-ray imaging and spectral data of unprecedented, subarcsecond, resolution. In Paper I we described the coordinates used in the event lists generated by the data processing system. In this paper we describe the boresight calibration process used to determine the mutual alignment of the aspect camera and the science instruments.

2 Making an observation

2.1 The Boresight Calibration

The Chandra aspect camera assembly (ACA) contains a primary and backup CCD detector, only one of which is used at a time. The ACA images the star field directly, and simultaneously observes images of fiducial lights attached to the X-ray instruments, which are visible via a periscope assembly. Images of the guide stars are obtained each second; a Kalman filter is used to combine the optical data with spacecraft gyro rates and provide a smoothed estimate of the aspect camera pointing direction and roll angle.

For each instrument, a correction is made to the aspect solution to take out misalignment between the instrument frame (the LSI frame, see Paper I) and the the aspect camera frame. This correction is calibrated using observations of celestial sources with known positions. The calibration depends on the adopted parameters of the instrument origins (in the STT frame, Paper I).

When an observation is performed, the spacecraft is given a set of target coordinates $C_t = (\alpha_t, \delta_t)$, a desired roll angle γ_t, Y and Z angular pointing offsets $dO = (0, dO_Y, dO_Z)$, and a SIM X and Z offset $dSIM_X, dSIM_Z$ relative to the instrument's default SIM position SIM_i.

The pointing offsets are applied to the target coordinates to get the ACA pointing direction $C_a = \alpha_a, \delta_a$. An additional offset known as the SI ALIGN matrix is also applied; the SI ALIGN matrix is different for each instrument and is a diagonal matrix (no rotations are introduced); we represent it by an additive vector dSI_i, where the subscript i indicates that the values are instrument-dependent. The values of the SI ALIGN matrix are chosen to place the target on a specific location on each detector. The pointing system locks up on appropriate guide stars and begins dithering the spacecraft around the ACA pointing direction with an amplitude of around 15 arcsec and a period of around 1000 seconds.

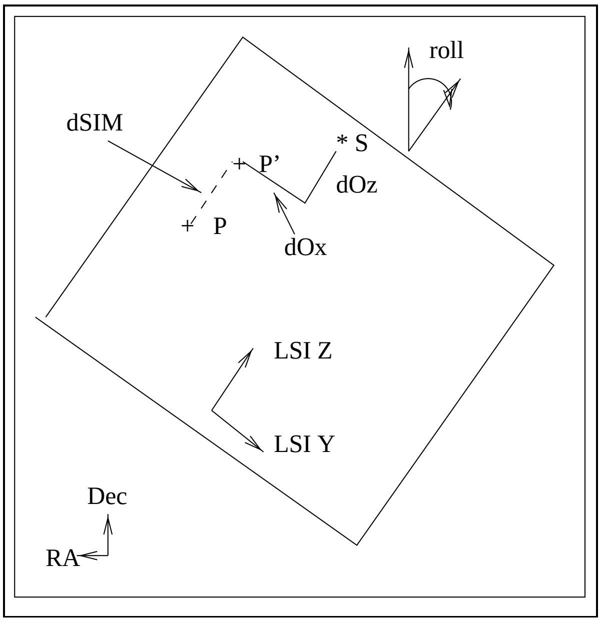

Figure 1: Aiming the telescope. A positive roll angle rotates the detector clockwise in the sky plane. A SIM offset moves the optical axis P' relative to the default aimpoint P, but does not move the target relative to the axis. The pointing offsets move the source coordinates relative to the target coordinates.

The pointing direction of ACA is then

$$C_a(t) = C_t + dSI_i + \mathrm{Rot}(\gamma_t)dO + dC_a(t)$$

where dC_a represents the time-dependent dither and where we introduce the notation

$$\mathrm{Rot}(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

for a 2D rotation matrix; when applied to a 3D vector it affects the Y and Z axes,

$$\mathrm{Rot}(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 \cos\theta & -\sin\theta & \\ 0 \sin\theta & \cos\theta & \end{pmatrix}$$

2.2 The Optical Axis and the Aimpoint

The point A in the X-ray (LSI or chip frame) image which corresponds to the RA and Dec of the instantaneous ACA pointing direction doesn't coincide with either the projection of the X-ray telescope optical axis or the location of the target X-ray source.

The location of the mean optical axis of the HRMA mirror, H, is found by observations of the point spread function as a function of position on the HRC-I detector. It is assumed to be offset from the ACA pointing direction by a fixed amount dC_H,

$$C_H(t) = C_a(t) + dC_H$$

In addition, each instrument has an 'aimpoint' P, a reference position on the instrument chips where the target is expected to land:

$$C_P(t) = C_a(t) - dSI_i$$

The SI ALIGN matrix is chosen so that C_P and C_t coincide when dO and dS are zero (no pointing offsets). We wish P and H to be close to each other so that targets will have a small point spread function. However, P may be chosen to be slightly different from H to avoid detector boundaries.

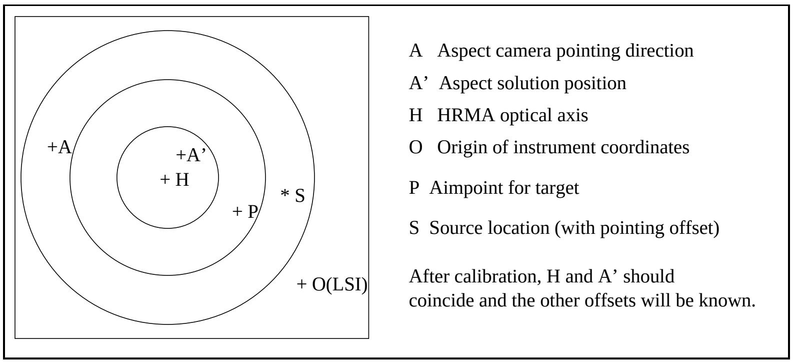

Figure 2: The various positions mentioned in the text. Moving the telescope moves A, A', H, P, O and the chips relative to S. Moving the SIM moves A,A',H,P and S relative to O and the chips.

2.3 Imaging the target

Consider a source at a general position C_S imaged by the HRMA. Let the operator T(), with inverse operator T'(), represent the mapping of an angular offset from the optical axis to a detected photon position in the focal coordinate (FC) system. This mapping involves a tangent plane projection followed by an intersection of the nominal incoming ray with the instrument chip planes.

Then the three-vector representing the focal coordinates of the detected source photons is

$$FC = T(\mathrm{Rot}(-\gamma_t)(C_S - C_H))$$

The projection operator and the rotation matrix commute.

We further assume that for a small offset dX,

$$T(X + dX) = T(X) + \Delta dX$$

which is just the statement that you can measure small coordinate offsets in the tangent plane image simply by converting millimeters (or pixels) to arcseconds. We assume that the plate scale Δ is already accurately known.

We now apply the equations of paper 1, but introduce an extra set of constant misalignments. Let the SIM translation axes be misaligned with respect to the focal coordinate axes by a rotation σ_0, and in addition let each instrument have an individual rotation λ_i with respect to the SIM translation axes.

Recall that (representing the equations of paper 1 in vector form)

$$FC = dX(t) + \mathrm{Rot}(\sigma_0 + \psi_X(t))STF$$

where the time-dependent terms are measured using the fiducial lights, and

$$STF = SIM_i + dSIM + OLSI + \mathrm{Rot}(\lambda_i)LSI$$

where we wish to determine the fixed origin offsets OLSI and misalignments λ_i. Note that $SIM_i + OLSI$ is always close to zero since the SIM aimpoints are chosen to place the instrument near the focus. Hence combining the above equations:

$$\begin{aligned} LSI &= \mathrm{Rot}(-\lambda_i)(STF - SIM_i - dSIM - OLSI) \\ &= \mathrm{Rot}(-\lambda_i - \sigma_0 - \psi_X(t))(FC - dX(t)) - \mathrm{Rot}(-\lambda_i)(SIM_i + dSIM + OLSI) \\ &= \mathrm{Rot}(-\lambda_i - \sigma_0 - \psi_X(t) - \gamma_t)T(C_S - C_H(t)) \\ &\quad -\mathrm{Rot}(-\lambda_i)(SIM_i + dSIM + OLSI + \mathrm{Rot}(-\sigma_0 - \psi_X(t))dX(t)) \end{aligned}$$

Hence, moving the SIM (dSIM nonzero) will translate the source along a line allowing you to determine the angle λ_i. Moving the pointing (C_H changes) allows you to determine the angle σ_0.

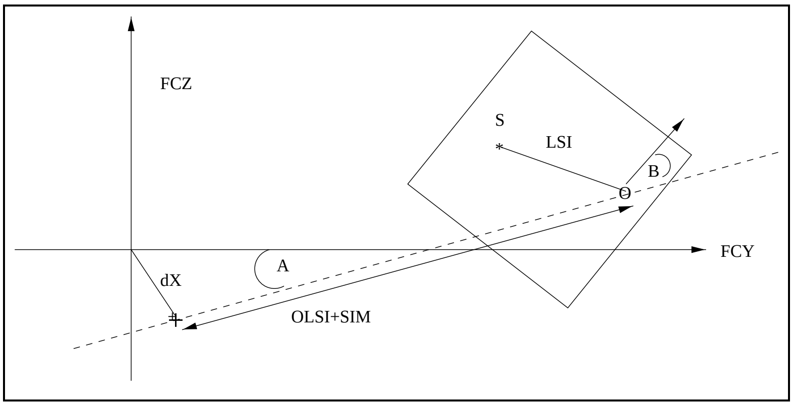

Figure 3: Focal plane misalignments (exaggerated). The STF (optical bench frame) origin is offset by dX from the focus due to time-dependent mechanical distortions. Optical bench motion is along the dashed line which is at an angle $A = \sigma_0 + \psi_X$ to the spacecraft axes. The instrument origin is offset from the STF origin by OLSI + SIM. The location of a pixel S relative to the instrument origin is given in LSI coordinates whose axes are at an angle $B = \lambda_i$ to the SIM motion direction.

3 Processing an observation

In the previous section we calculated the instrument-space coordinates of an imaged X-ray source of known celestial coordinates. In this section we present the method used to derive the celestial coordinates of the source in the data processing system, without assuming that the alignments are correctly calculated.

3.1 The corrected aspect solution and the CALALIGN matrix

The Chandra data system obtains the Kalman smoothed aspect solution and the fiducial light motion solution. The aspect solution is further corrected by the CALALIGN matrix, which introduces translations and rotations for each of the four standard aimpoints ACIS-I, ACIS-S, HRC-I, HRC-S. The corrected aspect solution celestial coordinates are then

$$C'_a = C_a + dC_i$$

and the roll is

$$\gamma_a = \gamma_t + d\gamma_i$$

These celestial coordinates correspond to a point A' on the detector. The intent is to calibrate the system so that A' = H, the optical axis.

3.2 The coordinates of a detected X-ray

Given the telemetered detection coordinates of the X-ray, we may unambiguously determine its raw LSI coordinates in three dimensions. Errors in the chip corner locations in the Y,Z plane are easily detected by inspection, and the pre-flight metrology and calibration of the out-of-plane tilt of the chips is assumed to be sufficiently accurate for determining celestial locations: to introduce a 0.1 arcsec error in the deduced position at the ends of the HRC-S (the detector with the largest off axis angles) requires a 0.2mm error in measuring the X offset of the chip ends, much larger than the expected uncertainty of a few microns.

However, there are significant uncertainties in the placement of the instrument on the optical bench. We must adjust raw LSI coordinates to corrected LSI' coordinates by applying an adopted alignment correction λ_i^e and then add in the adopted STT instrument origin $OLSI^e$. The superscript e denotes that these are our estimates of the values λ_i and $OLSI$ used in the previous section, and may not be equal to the actual values.

Next, we must correct to the STF frame by adding in the position of the SIM. The SIM position is determined by an encoder reading and converted to millimeters by a standard calibration. How repeatable and accurate is the SIM? We cannot easily distinguish between errors or variations in the SIM calibration and a thermal motion of the SIM or the entire ISIM (STF frame) with respect to the telescope, so we choose to assume a perfect SIM calibration and absorb any errors in the STF-to-FC transformation which is measured by the fiducial light solution. Another way of looking at this is to say that the STF frame is *defined* by the SIM calibration. I will express the telemetered SIM position as the sum of the standard SIM position for the instrument SIM_i and an offset dSIM. In general the telemetered value of dSIM is very close to the commanded one.

The estimated STF coordinates are then

$$STF^e = SIM_i + dSIM + \mathrm{Rot}(\lambda_i^e)LSI + OLSI^e$$

and the FC coordinates are

$$FC^e = dX(t) + \mathrm{Rot}(\sigma_0^e + \psi_X(t))STF^e$$

(assuming that the time-dependent variation dX and ψ_X is determined perfectly by the fid lights) The celestial coordinates are then found from

$$C_S^e = C'_a + \mathrm{Rot}(\gamma_a)T'(FC^e)$$

What is the error in our estimate of the celestial coordinates? We can substitute in our known expression for the true LSI value from the previous section.

$$STF^e = (1+\epsilon_i)STF - \epsilon_i(SIM_i + dSIM + OLSI) + (OLSI^e - OLSI)$$

where

$$\epsilon_i = \mathrm{Rot}(\lambda_i^e - \lambda_i) - 1$$

is a near-zero matrix giving our error in estimated instrument alignment. This equation says that our error in estimating the STF coordinates is equal to the fixed error in the instrument origin estimate plus an instrument misalignment term which grows as the SIM is moved from its standard position plus a misalignment term arising from the displacement of the photon itself. Substituting further in the expressions for estimated and true focal coordinates,

$$\begin{aligned} FC^e &= (1+\Sigma_i)FC + \Sigma_i dX(t) \\ &\quad + \mathrm{Rot}(\sigma_0^e + \psi_X(t))(-\epsilon_i(SIM_i + dSIM + OLSI) + (OLSI^e - OLSI)) \end{aligned}$$

where

$$\Sigma_i = \mathrm{Rot}(\sigma_0^e - \sigma_0 + \lambda_i^e - \lambda_i) - 1$$

is a near-zero matrix combining the instrument alignment and SIM alignment errors. Now we can project to the celestial sphere

$$\begin{aligned} C_S^e &= C_a' + \mathrm{Rot}(\gamma_a)T'(FC^e) \\ &= C_a + dC_i + \mathrm{Rot}(d\gamma_i)(1+\Sigma_i)(C_S - C_H) + \mathrm{Rot}(\gamma_a)\Delta\Sigma_i dX(t) - W_i + O_i \end{aligned}$$

where the term

$$W_i = \mathrm{Rot}(\gamma_a + \sigma_0^e + \psi_X(t))\Delta\epsilon_i(SIM_i + dSIM + OLSI)$$

is a celestial coordinate adjustment term arising from the instrument misalignments and may have a magnitude of a few arcseconds depending on the SIM position, and the term

$$O_i = \mathrm{Rot}(\gamma_a + \sigma_0^e + \psi_X(t))\Delta(OLSI^e - OLSI)$$

projects the instrument origin error onto the celestial sphere; its magnitude may be several arcminutes.

Note the time-dependent dX(t) term which contributes a small error to relative aspect reconstruction. Since the size of the misaligment rotations is determined to be less than 0.01 rad, and the amplitude of dX is of the order of 1 arcsecond, this term may be neglected. We will also neglect the small $\psi_X(t)$ terms.

In the boresight calibration we choose $d\gamma_i$ so that the term multiplying $(C_S - C_H)$ is unity, i.e.

$$d\gamma_i = -(\sigma_0^e - \sigma_0 + \lambda_i^e - \lambda_i)$$

Then the error in celestial coordinates is

$$C_S^e - C_S = dC_i - dC_H + W_i + O_i$$

In the boresight calibration calibration we choose dC_i to make the celestial errors disappear, so in fact

$$dC_i = dC_H - W_i - O_i$$

Thus, the adopted origin values $OLSI^e$ and the CALALIGN offsets dC_i are degenerate. By adopting an arbitrary $OLSI^e$ we have incorrect values of FC^e and thus incorrect off-axis angles and

DETX,DETY values but, thanks to CALALIGN, correct celestial coordinates as long as the term W_i remains constant (i.e. no SIM translation).

In order to obtain correct off-axis angles, we must determine the true OLSI, making the O_i term vanish (and choosing new values of dC_i). In order to obtain reliable celestial coordinates at all SIM positions, we must make W_i vanish by obtaining the correct values of the instrument misalignments λ_i. Determining σ_0 is not necessary for celestial coordinate accuracy as it can be absorbed in the definition of zero roll. That is, setting $\sigma_0 = 0$ redefines the spacecraft Y,Z axes. This may, however, be a problem for matching azimuthal features in the HRMA PSF.

4 Results for Chandra on-orbit boresight

The optical axis of Chandra was determined using HRC-I observations of the star HR 1099 (Markevitch 1999). The boresight was determined with observations of the open cluster NGC 2516 with each instrument. Details of the relevant analysis will be presented elsewhere.

For the boresight calibration we used the observations in table 1. The CAL ALIGN parameters used for the initial boresight (data processing up to early December 1999) and the improved boresight (implemented after Feb 2000) are given in table 2. CAL ALIGN also assumes a focal length of 10065.5117 mm.

Recent observations indicate a time dependent drift in at least the ACIS-S positions, possibly involving a 2" shift up to early Nov 1999 followed by a stable position. This shift is still under investigation.

Table 1: Observations used for analysis

Inst	OBSID	Target	Used for
HRC-I	1120-1152	HR 1099	Optical axis
ACIS-S	66	NGC 2516	Boresight
ACIS-I	65	NGC 2516	Boresight
HRC-S	68	NGC 2516	Boresight
ACIS-I	1278	PKS0312-77	Instrument offset
ACIS-I	469	PKS0312-77	Instrument offset
ACIS-S	1055	PKS0637-75	Instrument offset
ACIS-S	1235	Capella with HETG	Instrument offset
ACIS-S	474	PKS0637-75	Instrument offset
ACIS-S	1252	HR 1099 with HETG	Instrument offset
HRC-I	1211	HR 1099	Instrument offset
HRC-I	1296	LMC X-1	Instrument offset
HRC-I	1295	AR Lac	Instrument offset
HRC-S	1166	Capella	Instrument offset
HRC-S	62436	LMC X-1	Instrument offset

Table 2: Feb 2000 alignment matrices

Instrument	ACA ALIGN	FTS ALIGN

$$
\begin{array}{lcc}
\text{ACIS-S} & \begin{pmatrix} 1.0 & -0.00042385 & -0.000232987 \\ 0.000423213 & 1.0 & -0.0027337 \\ 0.0002341447 & 0.0027336 & 1.0 \end{pmatrix} & \begin{pmatrix} 1.0 & 0.000160413 & -0.0000749 \\ -0.000160194 & 1.0 & 0.0029229 \\ 0.0000754203 & -0.00292288 & 1.0 \end{pmatrix} \\
\text{ACIS-I} & \begin{pmatrix} 1.0 & -0.000430393 & -0.000231218 \\ 0.000429548 & 1.0 & -0.0036 \\ 0.000232784 & 0.0036 & 1.0 \end{pmatrix} & \begin{pmatrix} 1.0 & 0.000174783 & -0.00007235 \\ -0.00017445 & 1.0 & 0.0046 \\ 0.00007315 & -0.0046 & 1.0 \end{pmatrix} \\
\text{HRC-S} & \begin{pmatrix} 1.0 & -0.00043173 & -0.000262023 \\ 0.000431422 & 1.0 & -0.0012 \\ 0.00026253 & 0.0012 & 1.0 \end{pmatrix} & \begin{pmatrix} 1.0 & 0.000164897 & -0.0000990 \\ -0.000165138 & 1.0 & -0.0024 \\ 0.0000986 & 0.0024 & 1.0 \end{pmatrix} \\
\text{HRC-I} & \begin{pmatrix} 1.0 & -0.000413208 & -0.000238806 \\ 0.000413606 & 1.0 & 0.0017 \\ 0.000238117 & -0.0017 & 1.0 \end{pmatrix} & \begin{pmatrix} 1.0 & 0.000165798 & -0.0001286 \\ -0.000166149 & 1.0 & -0.0027 \\ 0.0001282 & 0.0027 & 1.0 \end{pmatrix}
\end{array}
$$

These alignment matrices are applied to the mirror-centered unit vector. We can therefore interpret the HRC-I alignment matrix as follows: there is a (LSIY,LSIZ) offset of (413, 238) microradians between the aspect frame and the HRC-I image frame, and a rotation of $\sin^{-1}(0.0017) = 0.1deg$. The fact that the values are different for different instruments reflects the remaining errors in the origin locations and in the values of λ_i.

Instrument	Offset (Arcsec)	Rotation (deg)
ACIS-S	87.43, 48.06	0.16
ACIS-I	88.77, 47.69	0.21
HRC-S	89.05, 54.04	0.07
HRC-I	85.23, 49.26	-0.10

We at least need to fix the rotations since these affect the celestial coordinates. Fixing the offsets is less important as that affects the calculated off axis angle slightly but nothing else. Since the optical axis was derived from HRC-I data One might propose a final universal ACA alignment matrix based on those values but with the rotation set to keep ACIS aligned with the SIM.

Table 4: Jan 2000 proposed alignment matrix

$$
\begin{array}{ll}
\text{Instrument} & \text{ACA ALIGN} \\
\text{All} & \begin{pmatrix} 1.0 & -0.000414 & -0.000238 \\ 0.000414 & 1.0 & -0.0040 \\ 0.000238 & 0.0040 & 1.0 \end{pmatrix}
\end{array}
$$

We would then recalculate instrument origins and rotations to match this alignment matrix. This reevaluation hasn't been done yet, and will probably wait until the drifts are better understood.

The positions of the fiducial lights are expressed in the corrected LSI coordinate system.

Table 5: Fiducial light positions, LSI coordinates

ACIS-S 1	25.40	-43.2982	86.3465
ACIS-S 2	25.40	39.2527	86.5668
ACIS-S 3	25.40	-0.3219	92.8507

ACIS-S 4	25.40	-102.5392	-6.4067
ACIS-S 5	25.40	90.5319	-6.0903
ACIS-S 6	25.40	-17.2820	-37.4259
ACIS-I 1	25.40	-43.2982	39.4665
ACIS-I 2	25.40	39.2527	39.6868
ACIS-I 3	25.40	-0.3219	45.9707
ACIS-I 4	25.40	-102.5392	-53.2867
ACIS-I 5	25.40	90.5319	-52.9703
ACIS-I 6	25.40	-17.2820	-84.3059
HRC-S 1	5.080	60.0	25.40
HRC-S 2	5.080	-60.0	25.40
HRC-S 3	5.080	60.0	-25.40
HRC-S 4	5.080	-60.0	-25.40
HRC-I 1	5.080	39.250	65.320
HRC-I 2	5.080	-39.250	65.760
HRC-I 3	5.080	60.860	-45.860
HRC-I 4	5.080	-60.860	-45.860

5 Summary

We have derived equations for the predicted instrument coordinates of a source in an X-ray telescope in the presence of offsets and misalignments. A method for deriving the instrument origins relative to the optical axis was presented, and we derive expressions for the residual misalignment and instrument origin error terms after boresighting as a function of optical bench position.

We thank Ian Evans and Janet DePonte for useful discussions, and the CXC Data System programmers and operators for software and processing support.

References

1. Markevitch, M., 1999, 'Optical Axis Measurement', CXC online internal memo dated 1999 Oct 11.