The birth-ultrafast-magnetic-field-decay model applied to isolated millisecond pulsars

Ricardo Heras

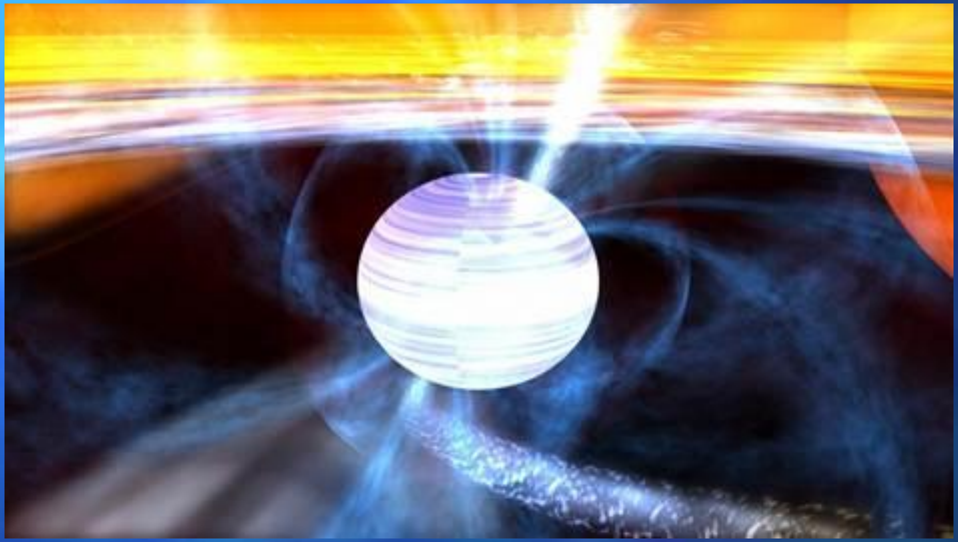
Preparatoria Abierta SEIEM Edo. Mexico

Millisecond pulsars MSPs

Pulsars with a rotational period in the range of about 1-20 milliseconds and magnetic fields in the range of 10^8- 10^9 G.

They are taught to begin life as longer period pulsars but are spun up through accretion in a binary relationship.

What about isolated MSPs?



This talk deals with three fundamental questions on isolated millisecond pulsars (IMSP's)

- Why do IMSP's have weaker magnetic fields compared to those of ordinary pulsars?

  $B \approx 2.59 \times 10^8$ G

- Why do IMSP's spin so rapidly?

  $P \approx .005$ s

- Why do IMSP's have transverse velocities smaller compared to those of ordinary pulsars?

  $V_t \approx 60.11$ km/s

(Average values of 9 IMP's whose transverse velocities have been reported)

Basic assumptions in the Birth-ultrafast-magnetic-field-decay model:

See R. Heras, "Pulsars are born as magnetars" in ASP conference 2012

During its birth process a neutron star experiences:

1. An increase of its period from the initial value P_0 to the current value P_s

(a change of rotational energy)

2. An exponential decay of its magnetic field from the initial value B_0 to the current surface value B_s

(a change of radiative energy)

3. An increase of its space velocity from the initial value V_0 to the current value V

(a change of kinetic energy)

Basic assumptions……

4. These birth energy changes are connected by

$$\underbrace{\frac{B_0^2 R^6 \ln(B_0/B_s)}{6c^3\tau^3}}_{\Delta E_{\rm rad}} + \underbrace{\frac{Mv^2}{2}}_{\Delta E_{\rm kin}} = \underbrace{\frac{4\pi^2 M R^2}{5}\left(\frac{1}{P_0^2} - \frac{1}{P_s^2}\right)}_{\Delta E_{\rm rot}};$$

where M and R are the radius and mass of the neutron star; c the speed of light and τ the characteristic time of the exponential field decay and the initial velocity is taken to be zero. **According to the green formula, the radiation loss and increase of kinetic energy are both at the expense of rotational energy.**

[A similar equation but with a different radiative term is the basis of the "Rocket Model" proposed by Harrison ad Tademaru, ApJ , 201, 447 (1975), See Eq. (12)]

Implications of the model:

For the **Crab** pulsar the equation yields $\tau \approx R/c$
if $P_0 \approx .019$ s and $B_0 = 5.8 \times 10^{15}$G

For the magnetar **J1809-1943** the equation gives $\tau \approx R/c$
if $P_0 = .02$ s and $B_0 = 9.5 \times 10^{15}$G

For the IMSP **B1257+12** the equation yields $\tau \approx R/c$
if $P_0 = .0059$ s and $B_0 = 2.7 \times 10^{15}$G

The characteristic time $\tau = R/c$ is consistent with the idea that all neutron stars are born with magnetic fields in the range of 10^{15}–10^{16}G and initial periods in range 1–20 ms

The time $\tau = R/c$ is the shortest theoretical time for a physical kick

Implications of the model……..

From the exponential law $B(t) = B_0 e^{-t/\tau}$ and $\tau = R/c$

It follows that $\tau_s = (R/c)\ln(B_0/B_s)$ is the time decay from B_0 to B_s. For field decays from one to eight orders of magnitude one has $2.3 \leqslant \ln(B_0/B_s) \leqslant 4.4$ and therefore $\tau_s \sim 10^{-4}$ s indicating an ultrafast magnetic field decay!

With $\tau = R/c$ the energy conversion takes the form

$$\boxed{\frac{B_0^2 R^3 \ln(B_0/B_s)}{6} + \frac{Mv^2}{2} = \frac{4\pi^2 M R^2}{5}\left(\frac{1}{P_0^2} - \frac{1}{P_s^2}\right)}$$

This formula is the fundamental equation of the birth-ultrafast-magnetic-field-decay model of neutron stars

Initial magnetic field of neutron stars

Using the canonical values

$$v \approx \sqrt{3/2}\, v_\perp,\ M = 1.4 M_\odot \text{ and } R = 10 \text{ km}$$

into the reed formula it implies

$$\boxed{B_0 = B_s e^{W([-\sigma v_\perp^2 + \lambda(P_0^{-2} - P_s^{-2})] B_s^{-2})/2}}$$

where $W(x)$ is the Lambert function, defined as the inverse of the function $f(x) = xe^x$ satisfying $W(x)e^{W(x)} = x.$

and $\sigma = \dfrac{9M}{R^3} = 2.52 \times 10^{16} \text{ gr/cm}^3$ and $\lambda = \dfrac{48 M \pi^2}{5R} \approx 2.65 \times 10^{29} \text{ gr/cm.}$

To apply the yellow formula for B_0 one needs to consider neutron stars whose $P_s, B_s \text{ and } v_\perp$ are known.

Application to IMSP's

IMSP	P_s	B_s	V_t
J0030+0451 4	4.865	2.00×10^8	65
J0711−6830	5.491	1.58×10^8	139
J1024−0719	5.162	3.16×10^8	45
J1453+1902	5.792	2.51×10^8	46
J1730−2304	8.123	3.98×10^8	53
J1744−1134	4.075	2.00×10^8	20
B1937+21	1.558	3.98×10^8	22
J2124−3358	4.931	2.51×10^8	67
J2322+2057	4.808	1.58×10^8	89

Averages

$B_s \approx 2.59 \times 10^8$ G

$P_s \approx .005$ s

$V_t \approx 60.66$ km/s

Application to IMSP's

The yellow formula and the average values yield the initial magnetic fields for the IMPs

$$B_0 \approx 1.3 \times 10^{16}\,G \quad \text{if} \quad P_0 \approx .004\ s$$

$$B_0 \approx 3.64 \times 10^{15}\,G \quad \text{if} \quad P_0 \approx .0049\ s$$

At the end of its formation, a neutron star may increase its rotational period from $P_0 \approx .0049$ s to $P_s \approx .005$ s during $\sim 10^{-4}$ s and then the rotational energy released can be transformed into kinetic and radiative energies in such a way that the IMPS acquires its transverse velocity $V_t \approx 60.66$ km/s provided it has the initial magnetic $B_0 \approx 3.64 \times 10^{15}$ G, which is in the range of magnetars.

Changing paradigms!

The generally accepted scenario of millisecond pulsar creation involves a long period of accretion in a low mass binary system. But Miller and Hamilton (2001) have proposed that the PSR 1257+12 was born with approximately its current period and magnetic field.

"some and perhaps all isolated millisecond pulsars may have been born with high spin rates and low magnetic fields instead of having been recycled by accretion."

The model proposed here predicts that very tiny fractions of a second after their formation, isolated millisecond pulsars display their observed small periods and low magnetic fields.

Birth magnetorotational instabilities!

Spruit (2008) has suggested that a differential rotation in the final stages of the core collapse process can produce magnetic fields typical of magnetars. Some form of magnetorotational instability may be the cause of an exponential growth of the magnetic field. "Once formed in core collapse, the magnetic field is in danger of decaying again by magnetic instabilities."

Geppert & Rheinhardt (2006) have discussed a magnetohydro dynamical process (MHD) that significantly reduces the initial magnetic field of a newly-born neutron star in fraccions of a second. "Such a field reduction is due to MHD-instabilities, which are inevitable if neutron stars are born as magnetars".

Birth loss of rotational energy!

The idea of a loss of rotational energy during the birth process of millisecond pulsars was already considered by Usov (1992). "Once formed such rapidly rotating and strongly magnetized neutron stars [$\sim 10^{15}$ G] would lose their rotational energy catastrophically, on a timescale of seconds or less."

Final comment:

The model proposed here also applies to other families of NS such as magnetars or radio pulsars. The idea that all NS are born with magnetic fields typical of magnetars and periods typical of millisecond pulsars accounts for a unification of NS.

Answers to the initial questions

- **Why do IMSP's have weaker magnetic fields compared to those of ordinary pulsars? B ≈ 2.59 x 10^8 G**
  At the end of their formation, they experienced a sudden decay of their initial magnetic fields ~ 10^{15} -10^{16} G. The associated loss of radiation is at the expense of a loss of rotational energy.

- **Why do IMSP's spin so rapidly? P ≈ .005 s**
  They are born with high spin rates. At the end of its formation they undergo a small increase of their initial rotation period.

- **What is the origin of the transverse velocities of IMSP's?**
  At the end of they formation, they acquire their proper motion at the expense of a loss of rotational energy.

The answers are given in the context of birth magnetorotational instabilities!

The Larmor formula for the power radiated by a time-varying magnetic dipole moment $P = 2\ddot{\mu}^2/(3c^3)$ and the estimate $\ddot{\mu} \sim \mu_0/\tau^2$, where τ is the characteristic time in the exponential field decay law $B(t) = B_0 e^{-t/\tau}$, imply the equation $P \simeq 2\mu_0^2/(3c^3\tau^4)$, which can be used together with the relation $\mu_0 = B_0 R^3/2$ to yield the power radiated by a neutron star of radius R and an initial magnetic field B_0:

Mathematical calculation

$$P \simeq \frac{B_0^2 R^6}{6c^3\tau^4}.$$

$$\tau_s = \tau \ln(B_0/B_s).$$

$$E_{\rm rad} \simeq \tau_s P$$

$$E_{\rm rad} \simeq \frac{B_0^2 R^6 \ln(B_0/B_s)}{6c^3\tau^3}$$

$$\tau = R/c.$$

$$\boxed{E_{\rm rad} \simeq \frac{B_0^2 R^3 \ln(B_0/B_s)}{6}}$$

Radiative Energy

Thank you for your time!

e-mail: ricardoherasosorno@gmail.com

website: http://www.ricardoheras.com

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