Solving the Riemann Problem for Realistic Astrophysical Fluids

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Motivation

The need to incorporate realistic equations of state (EOS) in (radiation-)hydrodynamics is becoming increasingly important in computational astrophysics. The state-of-the-art of many numerically intensive research problems that include radiation transfer in such diverse sub-fields as accretion disks, binary mergers, star formation, supernovae, are approaching the state where more accurate EOS relating fluid state variables (e.g. density, pressure and temperature) is necessary. Commonly used Riemann solvers assume perfect EOS $\epsilon = p/[\rho(\gamma-1)]$.

Objective

1. We present methods to solve the Riemann problem exactly for realistic gases, with realistic EOS.
2. The solution of the exact EOS Riemann solver and the solution of the original exact Riemann solver match when calculating perfect gas Euler equations.
3. The solution of the new Harten-Lax-van Leer-Contact (HLLC) general EOS Riemann solver approaches the exact solution.
4. The HLLC general EOS Riemann solver is efficient and can be applied to real astrophysical problems.

1D Euler equations and Riemann problem

We are interested in the general EOS Euler equations.

$$\begin{cases} \rho_t + (\rho u)_x = 0 & (1) \\ (\rho u)_t + (\rho u^2 + p)_x = 0 & (2) \\ E_t + [(E+p)u]_x = 0 & (3) \end{cases}$$

where,

$$E = \rho(u^2/2 + \epsilon) \tag{4}$$

$$\epsilon = \epsilon(\rho, p) \quad \text{or} \quad p = p(\rho, \epsilon) \tag{5}$$

Riemann solver is aimed to solve the quasi-linear approximate hyperbolic system of the Euler equations.

$$\mathbf{U}_t + \mathbf{A}(\mathbf{U})\mathbf{U}_x = 0 \tag{6}$$

where

$$\mathbf{A} = \partial \mathbf{F}/\partial \mathbf{U} \tag{7}$$

$$\mathbf{U} = [\rho, \rho u, E]^T \tag{8}$$

$$\mathbf{F} = [\rho u, \rho u^2 + p, u(E+p)]^T \tag{9}$$

The EOS of pure hydrogen gas

Γ is the ratio of adiabatic sound speed to isothermal sound speed,

$$\Gamma = c_s/c_T \tag{10}$$

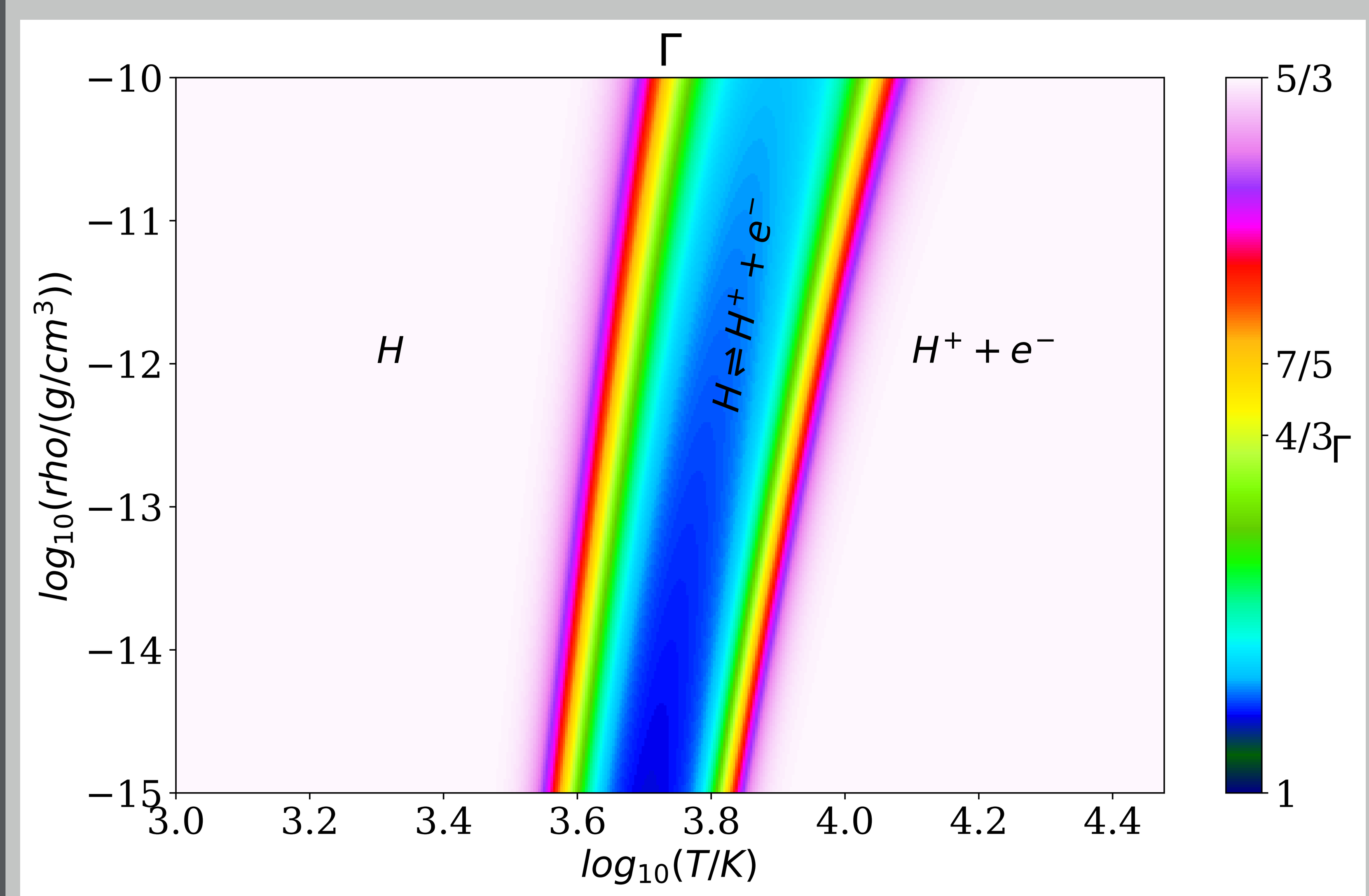

Figure 1: Γ drops to around 1.06 when hydrogen is ionizing.

Numerical Results: phase transition presents

A rightward moving shock tube test. The solution consists of a left rarefaction wave, a contact discontinuity and a rightward moving shock. The initial states are distinguished by $x = 0$.

ρ_L (g·cm^{-3})	v_L (km·s^{-1})	T_L (K)	ρ_R (g·cm^{-3})	v_R (km·s^{-1})	T_R (K)
10^{-13}	0	20000	10^{-15}	0	3000

Table 1: Initial left and right states.

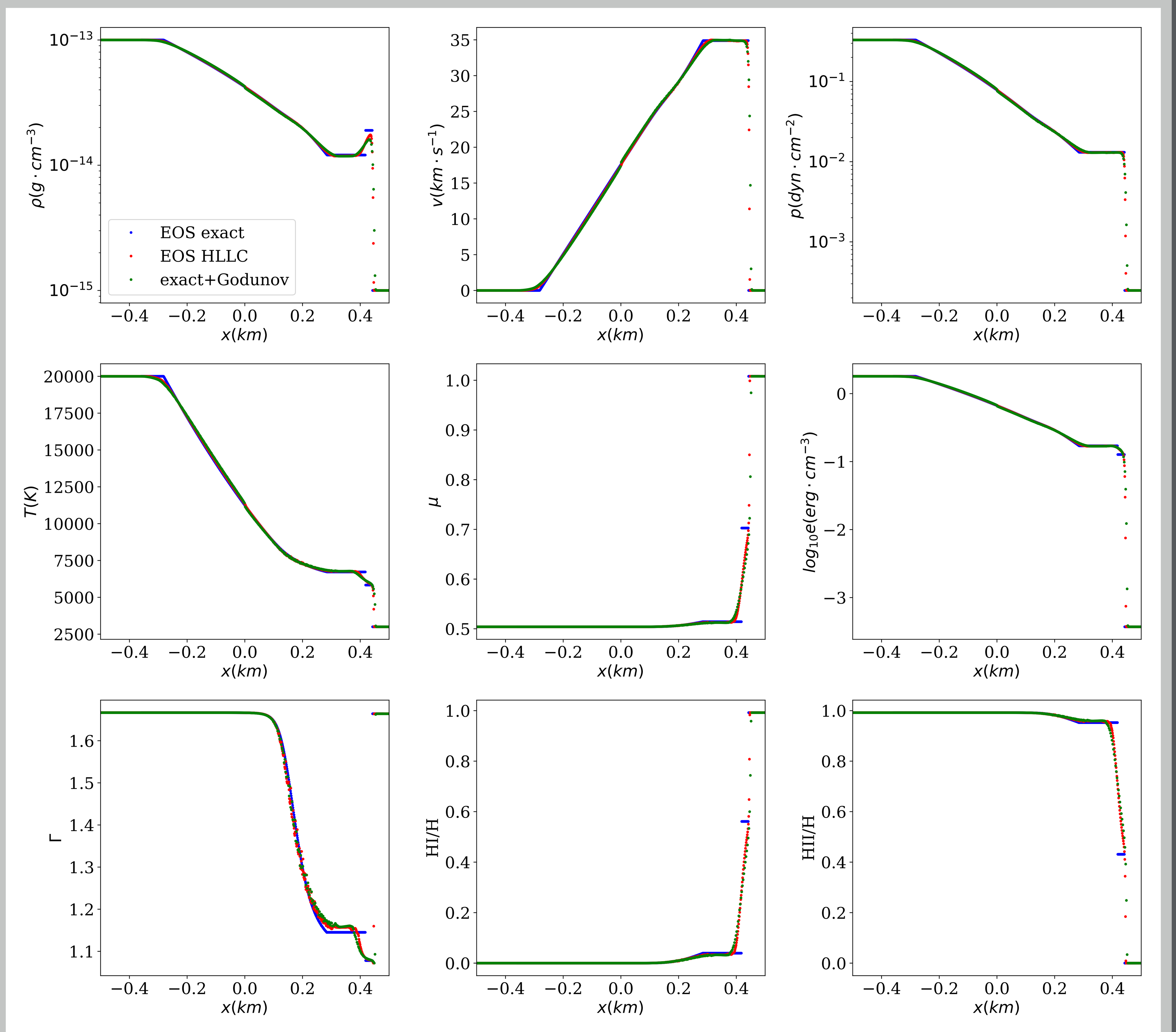

Figure 2: The solutions at $dt = 0.012$s. The blue dots show the results from the exact general EOS Riemann solver and the red dots show the results from the HLLC general EOS Riemann solver with $N = 800$. The green dots show the results from the exact general EOS Riemann solver with Godunov scheme and $N = 400$. The The panels in the first row from the left to the right are the density ρ in g·cm^{-3}, the velocity v in km·s^{-1}, and the pressure p in dyn·cm^{-2}. The panels in the second row from the left to the right are the temperature T in K, the mean atomic weight μ, and the specific internal energy ϵ in log_{10} erg·cm^{-3}. The panels in the third row from the left to the right are the ratio of adiabatic sound speed to the isothermal sound speed Γ, the fraction of atomic hydrogen, and the fraction of the ionized hydrogen. All the x coordinates are in km. The approximate solver gives an overall satisfactory result as it capture the position of the shock, the contact discontinuity and the starting point and the endpoint of the rarefaction wave. The temperature and the ionization fraction which are of great importance to radiation-hydrodynamics, are also resolved correctly.

The efficient new HLLC general EOS Riemann solver can give satisfactory solution when there is a phase transition.

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